

Imaging System Properties

For imaging system with real world image $f(x, y)$, focal plane image $g(x, y)$, and magnification M

$$\begin{aligned} g(x, y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\xi, \eta) h(x - M\xi, y - M\eta) d\xi d\eta \\ &= \frac{1}{M^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f\left(\frac{\xi}{M}, \frac{\eta}{M}\right) h(x - \xi, y - \eta) d\xi d\eta \end{aligned}$$

Define the function

$$\tilde{f}(x, y) \triangleq f\left(\frac{\xi}{M}, \frac{\eta}{M}\right)$$

- Notice that $\tilde{f}(x, y)$ is just a smaller version of $f(x, y)$.

In the frequency domain,

$$G(u, v) = \tilde{F}(u, v) \frac{1}{M} H(u, v)$$

$$\begin{aligned} g(x, y) &\overset{CSFT}{\longleftrightarrow} G(u, v) \\ h(x, y) &\overset{CSFT}{\longleftrightarrow} H(u, v) \\ \tilde{f}(x, y) &\overset{CSFT}{\longleftrightarrow} \tilde{F}(u, v) \end{aligned}$$

Optical System Transfer Functions

The *Optical Transfer Function (OTF)* is defined as:

$$\frac{H(u, v)}{H(0, 0)}$$

The *Modulation Transfer Function (MTF)* is defined as:

$$\left| \frac{H(u, v)}{H(0, 0)} \right|$$

The *Point Spread Function (PSF)* is defined as:

$$h(x, y)$$

Notice that when $f(x, y) = \delta(x, y)$

$$\begin{aligned} g(x, y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(\xi, \eta) h(x - M\xi, y - M\eta) d\xi d\eta \\ &= h(x, y) \end{aligned}$$