Color

- What is color?
  - Color is a human perception (a percept).
  - Color is not a physical property...
  - But, it is related the the light spectrum of a stimulus.
- Can we measure the percept of color?
  - Semantic names - red, green, blue, orange, yellow, etc.
  - These color semantics are largely culturally invariant, but not percise.
  - Currently, knowone has a accurate model for predicting perceived color from the light spectrum of a stimulus.
  - Currently, noone has an accurate model for predicting the percept of color.
- Can we tell if two colors are the same?
  - Two colors are the same if they match at *all* spectral wavelengths.
  - However, we will see that two colors are also the same if they match on a 3 dimensional subspace.
  - The values on this three dimensional subspace are called *tristimulus* values.
  - Two colors that match are called *metamers*.

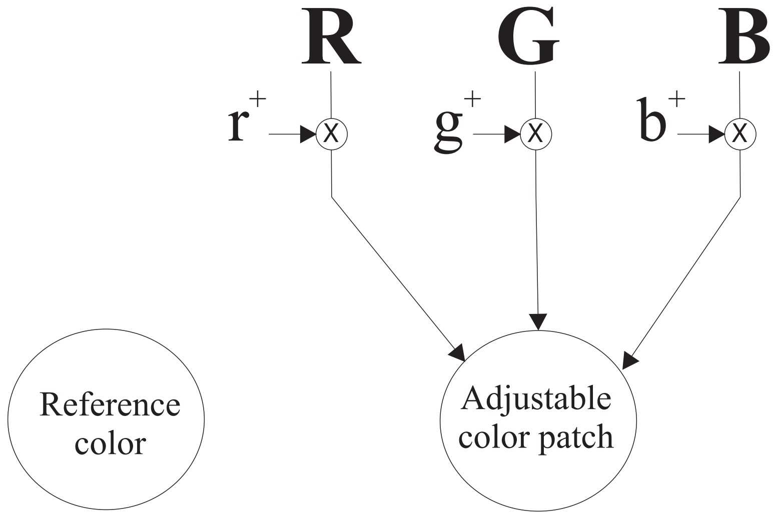
Matching a Color Patch

- Experimental set up:
  - Form a reference color patch with a known spectral distribution $I(\lambda)$.
  - Form a second adjustible color patch by adding light with three different spectral distributions $I_r(\lambda)$, $I_g(\lambda)$, and $I_b(\lambda)$.
  - Control the amplitude of each component with three individual positive constants r^+, g^+, and b^+.
  - The total spectral content of the adjustible patch is then

$$r\, I_r(\lambda) + g\, I_g(\lambda) + b\, I_b(\lambda) \ .$$

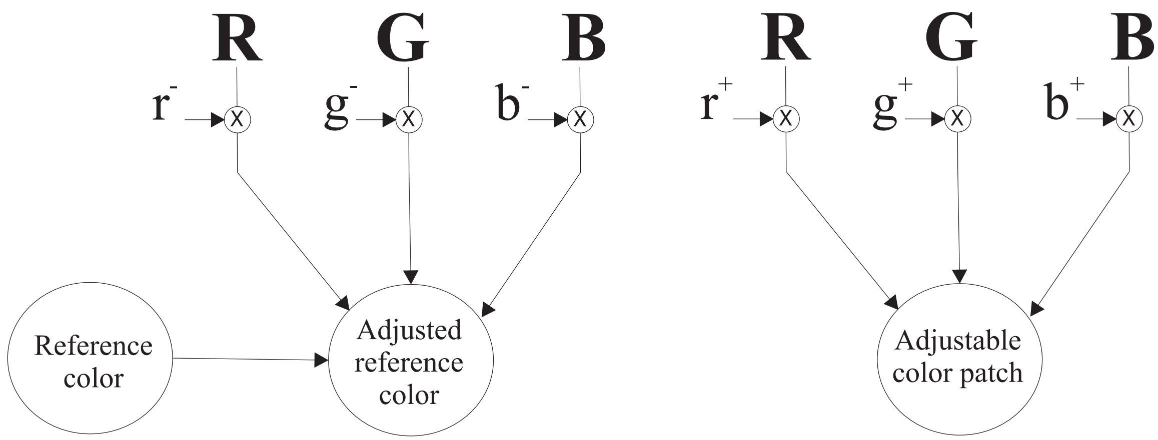
- Choose (r, g, b) to match the two color patches.

Simple Color Matching with Primaries



- Choose (r^+, g^+, b^+) to match the two color patches.
- The values of (r, g, b) must be positive!
- Definitions:
  - **R**, **G**, and **B** are known as color primaries.
  - r^+, g^+, and b^+ are known as tristimulus values.
- Problem:
  - Some colors can not be matched, because they are too "saturated".
  - These colors result in values of r^+, g^+, or b^+ which are 0.
  - How can we generate negative values for r^+, g^+, or b^+?

Improved Color Matching with Primaries



- Add color primaries to reference color!
- This is equivalent to subtracting them from adjustable patch.
- Equivalent tristimulus values are:

$$\begin{aligned} r &= r^+ - r^- \\ g &= g^+ - g^- \\ b &= b^+ - b^- \end{aligned}$$

- In this case, r, g, and b can be both positive and negative.
- All colors may be matched.

Grassman's Law

- Grassman's law: Color perception is a 3 dimensional linear space.
- Superposition:
  - Let $I_1(\lambda)$ have tristimulus values (r_1, g_1, b_1), and let $I_2(\lambda)$ have tristimulus values (r_2, g_2, b_2).
  - Then $I_3(\lambda) = I_1(\lambda) + I_2(\lambda)$ has tristimulus values of

$$(r_3, g_3, b_3) = (r_1, g_1, b_1) + (r_2, g_2, b_2)$$

- This implies that tristimulus values can be computed with a general linear operators of the form

$$\begin{aligned} r &= \int_0^\infty r_0(\lambda)\, I(\lambda) d\lambda \\ g &= \int_0^\infty g_0(\lambda)\, I(\lambda) d\lambda \\ b &= \int_0^\infty b_0(\lambda)\, I(\lambda) d\lambda \end{aligned}$$

  for some functions $r_0(\lambda)$, $g_0(\lambda)$, and $b_0(\lambda)$.
- Definition: $r_0(\lambda)$, $g_0(\lambda)$, and $b_0(\lambda)$ are known as color matching functions.

Measuring Color Matching Functions

- Consider when $I(\lambda) = \delta(\lambda - \lambda_o)$ is a line spectrum.

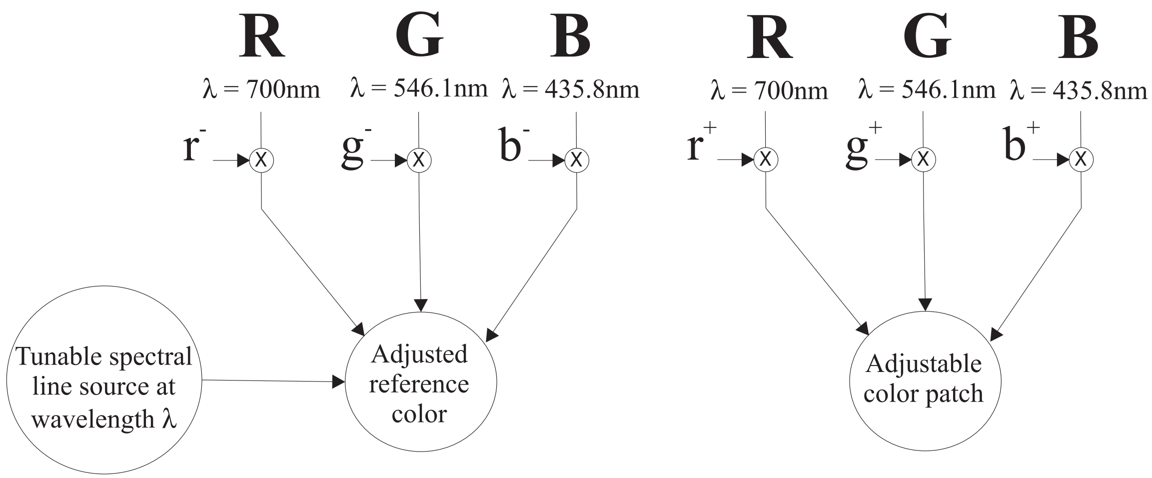
$$
\begin{aligned}
r &= \int_0^\infty r_0(\lambda)\,\delta(\lambda - \lambda_0)d\lambda = r_o(\lambda) \\
g &= \int_0^\infty g_0(\lambda)\,\delta(\lambda - \lambda_0)d\lambda = g_o(\lambda) \\
b &= \int_0^\infty b_0(\lambda)\,\delta(\lambda - \lambda_0)d\lambda = b_o(\lambda)
\end{aligned}
$$

- Solution:
  - Color match to a reference color generated by a pure spectral source.
  - A tunable laser can be used to generate a reference color.

CIE Standard RGB Color Matching Functions

- An organization call CIE (Commission Internationale de l'Eclairage) defined all practical standards for color measurements (colorimetery).
- CIE 1931 Standard 2^o Observer:
  - Uses color patches that subtended 2^o of visual angle.
  - $\mathbf{R}, \mathbf{G}, \mathbf{B}$ color primaries are defined by pure line spectra (delta functions in wavelength) at 700nm, 546.1nm, and 435.8nm.
  - Reference color is a spectral line at wavelength λ.
- CIE 1965 10^o Observer: A slightly different standard based on a 10^o reference color patch and a different measurement technique.

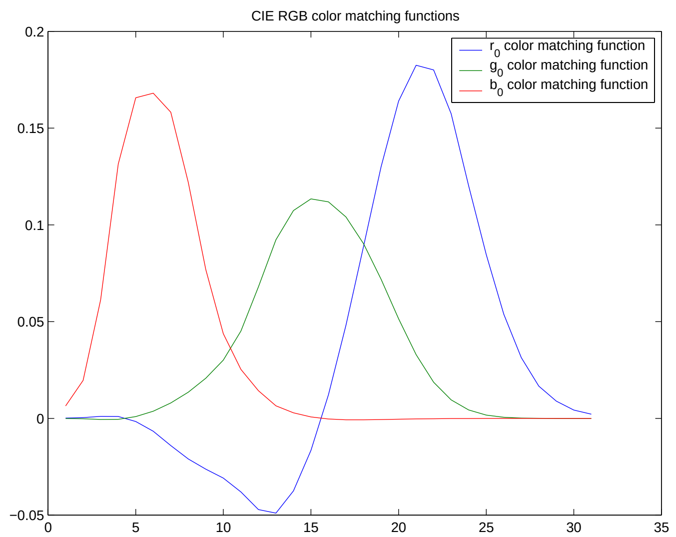
RGB Color Matching Functions for CIE Standard 2^o Observer



- The color matching functions are then given by

$$\begin{aligned} r_o(\lambda) &= r^+ - r^- \\ g_o(\lambda) &= g^+ - g^- \\ b_o(\lambda) &= b^+ - b^- \end{aligned}$$

where λ is the wavelength of the reference line spectrum.



Review of Colorimetry Concepts

1. $\mathbf{R}, \mathbf{G}, \mathbf{B}$ are color primaries used to generate colors.
2. (r, g, b) are tristimulus values used as weightings for the primaries.

$$\text{Color} = [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

3. $(r_0(\lambda), g_0(\lambda), b_0(\lambda))$ are the color matching functions used to compute the tristimulus values.

$$\begin{aligned} r &= \int_0^\infty r_0(\lambda)\, I(\lambda) d\lambda \\ g &= \int_0^\infty g_0(\lambda)\, I(\lambda) d\lambda \\ b &= \int_0^\infty b_0(\lambda)\, I(\lambda) d\lambda \end{aligned}$$

Problems with CIE RGB

- Some colors generate negative values of (r, g, b).
- This results from the fact that the color matching functions $r_0(\lambda)$, $g_0(\lambda)$, $b_0(\lambda)$ can be negative.
- The color primaries corresponding to CIE RGB are very difficult to reproduce. (pure spectral lines)
- Partial solution: Define new color matching functions $x_0(\lambda)$, $y_0(\lambda)$, $z_0(\lambda)$ such that:
  - Each function is positive
  - Each function is a linear combination of $(r_0(\lambda), g_0(\lambda), b_0(\lambda))$.

CIE XYZ Definition

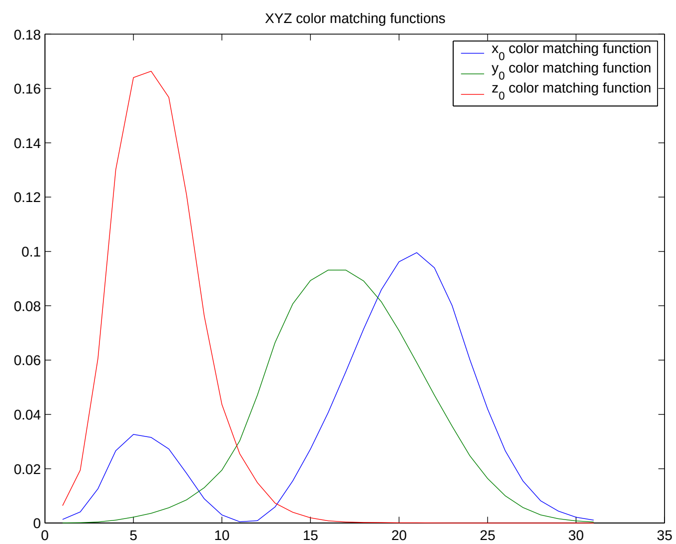
- CIE XYZ in terms of CIE RGB so that

$$\begin{bmatrix} x_0(\lambda) \\ y_0(\lambda) \\ z_0(\lambda) \end{bmatrix} = \mathbf{M} \begin{bmatrix} r_0(\lambda) \\ g_0(\lambda) \\ b_0(\lambda) \end{bmatrix}$$

  where

$$\mathbf{M} = \begin{bmatrix} 0.490 & 0.310 & 0.200 \\ 0.177 & 0.813 & 0.010 \\ 0.000 & 0.010 & 0.990 \end{bmatrix}$$

- This transformation is chosen so that $x_0(\lambda) \geq 0$, $y_0(\lambda) \geq 0$, and $z_0(\lambda) \geq 0$.



XYZ Color Transformations

- The XYZ tristimulus values may then be calculated as:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \int_0^\infty \begin{bmatrix} x_0(\lambda) \\ y_0(\lambda) \\ z_0(\lambda) \end{bmatrix} I(\lambda) d\lambda = \int_0^\infty \mathbf{M} \begin{bmatrix} r_0(\lambda) \\ g_0(\lambda) \\ b_0(\lambda) \end{bmatrix} I(\lambda) d\lambda = \mathbf{M} \int_0^\infty \begin{bmatrix} r_0(\lambda) \\ g_0(\lambda) \\ b_0(\lambda) \end{bmatrix} I(\lambda) d\lambda$$

- So we have that

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{M} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

- RGB can be computed from XYZ as:

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \mathbf{M}^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- Comments:
  - Always use upper case letters for XYZ!
  - Y value represents luminance component of image

XYZ Color Primaries

- The XYZ color primaries are computed as

$$\text{Color} = [\mathbf{X}, \mathbf{Y}, \mathbf{Z}] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} r \\ g \\ b \end{bmatrix} = [\mathbf{R}, \mathbf{G}, \mathbf{B}]\, \mathbf{M}^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- So, theoretically

$$[\mathbf{X}, \mathbf{Y}, \mathbf{Z}] = [\mathbf{R}, \mathbf{G}, \mathbf{B}]\, \mathbf{M}^{-1}$$

where

$$\mathbf{M}^{-1} = \begin{bmatrix} 1.3644 & -0.8958 & -0.4686 \\ -0.5148 & 1.4252 & 0.0896 \\ 0.0052 & -0.0144 & 1.0092 \end{bmatrix}$$

- Problem: Negative values of RGB mean that XYZ primaries can not be realized from physical combinations of CIE RGB.
- Fact: XYZ primaries are imaginary! They can not be realized physically.

Chromaticity Coordinates

- Tristimulus values X, Y, Z specify a color's:
  - Lightness - light or dark
  - Huge - red, orange, yellow, green, blue, purple
  - Saturation - pink-red; pastel-flouresent; baby blue-deep blue
- The *chromaticity* specifies the huge and saturation, but not the lightness.

$$
\begin{aligned}
x &= \frac{X}{X+Y+Z} \\
y &= \frac{Y}{X+Y+Z} \\
z &= \frac{Z}{X+Y+Z}
\end{aligned}
$$

- Properties of chromaticity coordinates
  - $x + y + z = 1$ - Third compent can always be computed from first two.
  - Typically, (x, y) are specified
  - Striaght lines in XYZ map to straight lines in (x, y).

Projection Property of Chromaticity Coordinates

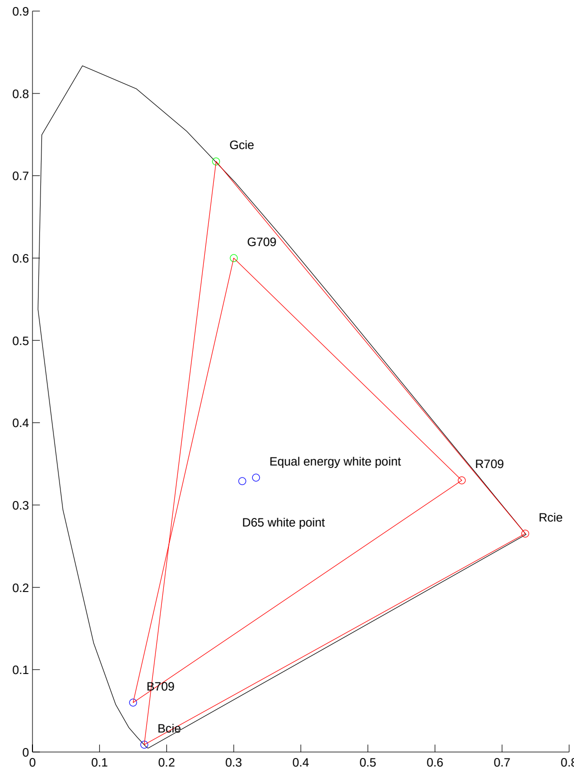
- Let X_1, Y_1, Z_1 and X_2, Y_2, Z_2 be two different colors and let

$$(X_3, Y_3, Z_3) = \alpha(X_1, Y_1, Z_1) + \beta(X_2, Y_2, Z_2)$$

then

$$\begin{aligned}
(x_3, y_3) &= \left(\frac{\alpha X_1 + \beta X_2}{X_3 + Y_3 + Z_3}, \frac{\alpha Y_1 + \beta Y_2}{X_3 + Y_3 + Z_3}\right) \\
&= \left(\frac{\alpha X_1}{X_3 + Y_3 + Z_3}, \frac{\alpha Y_1}{X_3 + Y_3 + Z_3}\right) + \left(\frac{\beta X_2}{X_3 + Y_3 + Z_3}, \frac{\beta Y_2}{X_3 + Y_3 + Z_3}\right) \\
&= \frac{X_1 + Y_1 + Z_1}{X_3 + Y_3 + Z_3}\left(\frac{\alpha X_1}{X_1 + Y_1 + Z_1}, \frac{\alpha Y_1}{X_1 + Y_1 + Z_1}\right) \\
&\quad + \frac{X_2 + Y_2 + Z_2}{X_3 + Y_3 + Z_3}\left(\frac{\beta X_2}{X_2 + Y_2 + Z_2}, \frac{\beta Y_2}{X_2 + Y_2 + Z_2}\right) \\
&= \alpha\frac{X_1 + Y_1 + Z_1}{X_3 + Y_3 + Z_3}(x_1, y_1) + \beta\frac{X_2 + Y_2 + Z_2}{X_3 + Y_3 + Z_3}(x_2, y_2) \\
&= \alpha'(x_1, y_1) + \beta'(x_2, y_2)
\end{aligned}$$

Chromaticity Diagrams



- Linear combinations of colors form straight lines
- Horse shoe shape results form XYZ color matching functions
- Straight line connecting red and blue is referred to as "line of purples"
- RGB primaries form triangular color gammut
- Center is white

What is White Point?

- What is white point:
  - Absolute scaling of (r, g, b) values required for calibrated image data. This determines the color associated with $(r, g, b) = (1, 1, 1)$.
  - Color of illuminant in scene. By changing white point, one can partially compensate for changes due to illumination color. (camcorders)
  - Color of paper in printing applications. Color of paper is brightest white usually possible. Should a color photocopier change the color of the paper? Usually no.

Defining White Point?

- Ideally white point specifies the spectrum of the color white.

$$I_w(\lambda)$$

- This turn specifies XYZ coordinates

$$\begin{aligned}
X_w &= \int_0^\infty x_0(\lambda)\, I_w(\lambda) d\lambda \\
Y_w &= \int_0^\infty y_0(\lambda)\, I_w(\lambda) d\lambda \\
Z_w &= \int_0^\infty z_0(\lambda)\, I_w(\lambda) d\lambda
\end{aligned}$$

which in turn specifies chromaticity components

$$\begin{aligned}
x_w &= \frac{X_w}{X_w + Y_w + Z_w} \\
y_w &= \frac{Y_w}{X_w + Y_w + Z_w}
\end{aligned}$$

- Comments
  - White point is usually specified in chromaticity.
  - Knowing (x_w, y_w) does not determine $I_w(\lambda)$.

Typical White Points

- Equal energy white:

$$
\begin{aligned}
I_{EE}(\lambda) &= 1 \\
(x_{EE}, y_{EE}) &= (1/3, 1/3)
\end{aligned}
$$

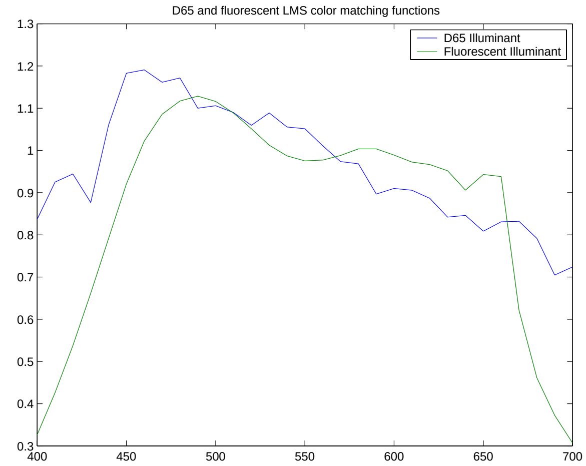
- D65 illuminant (specified for PAL):

$$
\begin{aligned}
I_{65}(\lambda) &= \text{Natural Sun Light} \\
(x_{65}, y_{65}) &= (0.3127, 0.3290)
\end{aligned}
$$

- C illuminant (specified for NTSC):

$$
\begin{aligned}
I_{c}(\lambda) &= \text{not defined} \\
(x_{65}, y_{65}) &= (0.310, 0.316)
\end{aligned}
$$

- Comments:
  - Equal energy white is not commonly used.
  - C was the original standard for NTSC video.
  - $D65$ has become the dominant standard.
  - $D65$ corresponds to a color temperature of 6500^oK.



White Point Correction

- Standard color matching functions assume equal energy white point
  - Any standard color matching function assumes unit area normalization.

$$
\begin{aligned}
1 &= \int_0^\infty r_0(\lambda) d\lambda \\
1 &= \int_0^\infty g_0(\lambda) d\lambda \\
1 &= \int_0^\infty b_0(\lambda) d\lambda
\end{aligned}
$$

  - Therefore:

$$I_{EE}(\lambda) = 1 \Rightarrow (r, g, b) = (1, 1, 1)$$

- White point corrected/gamma corrected data is compute as:

$$
\begin{aligned}
\tilde{r} &\triangleq \left(\frac{r}{r_{wp}} \right)^{1/\gamma} \\
\tilde{g} &\triangleq \left(\frac{g}{g_{wp}} \right)^{1/\gamma} \\
\tilde{b} &\triangleq \left(\frac{b}{b_{wp}} \right)^{1/\gamma}
\end{aligned}
$$

  - So,

$$(\tilde{r}, \tilde{g}, \tilde{b}) = (1, 1, 1) \Rightarrow (r, g, b) = (r_{wp}, g_{wp}, b_{wp})$$

    where (r_{wp}, g_{wp}, b_{wp}) is the desired white point.

Typical RGB Color Primaries

- NTSC 601 standard primaries:

$$
\begin{aligned}
(x_r, y_r) &= (0.67, 0.33) \\
(x_g, y_g) &= (0.21, 0.71) \\
(x_b, y_b) &= (0.14, 0.08)
\end{aligned}
$$

  - These color primaries are not typically used anymore.

- PAL standard primaries:

$$
\begin{aligned}
(x_r, y_r) &= (0.64, 0.33) \\
(x_g, y_g) &= (0.29, 0.60) \\
(x_b, y_b) &= (0.15, 0.06)
\end{aligned}
$$

  - PAL is the TV standard used in Europe

- Rec. 709 standard primaries:

$$
\begin{aligned}
(x_r, y_r) &= (0.5541, 0.2857) \\
(x_g, y_g) &= (0.3000, 0.6000) \\
(x_b, y_b) &= (0.1500, 0.0600)
\end{aligned}
$$

  - More saturated then 601 primaries.
  - Most commonly used primary colors for display monitors and TV's.

Example: 601 Color Primaries With EE White Point

- Find a transformation $\mathbf{M}$ so that

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{M} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

  - Columns of $\mathbf{M}$ are proportional to color primaries.
  - Rows of $\mathbf{M}$ sum to 1 $\Rightarrow$ equal energy white point.

- Solve the equation

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.67 & 0.21 & 0.14 \\ 0.33 & 0.71 & 0.08 \\ 0.00 & 0.08 & 0.78 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

- This results in $[\alpha_1, \alpha_2, \alpha_3] = (0.9867, 0.8148, 1.1985)$, and

$$\mathbf{M} = \begin{bmatrix} 0.67 & 0.21 & 0.14 \\ 0.33 & 0.71 & 0.08 \\ 0.00 & 0.08 & 0.78 \end{bmatrix} \begin{bmatrix} 0.9867 & 0 & 0 \\ 0 & 0.8148 & 0 \\ 0 & 0 & 1.1985 \end{bmatrix} = \begin{bmatrix} 0.6611 & 0.1711 & 0.1678 \\ 0.3256 & 0.5785 & 0.0959 \\ 0 & 0.0652 & 0.9348 \end{bmatrix}$$

Example: 601 Color Primaries With C White Point

- Find a transformation $\mathbf{M}$ so that

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{M} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

  where

  - Columns of $\mathbf{M}$ are proportional to color primaries.
  - Rows of $\mathbf{M}$ sum to $[0.310, 0.316, 0.374] \times \mathit{constant}$.
  - Middle rows of $\mathbf{M}$ sum to $1 \Rightarrow$ unit luminance.

- Solve the equation

$$\frac{1}{0.316} \begin{bmatrix} 0.310 \\ 0.316 \\ 0.374 \end{bmatrix} = \begin{bmatrix} 0.9810 \\ 1 \\ 1.1835 \end{bmatrix} = \begin{bmatrix} 0.67 & 0.21 & 0.14 \\ 0.33 & 0.71 & 0.08 \\ 0.00 & 0.08 & 0.78 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

- This results in $[\alpha_1, \alpha_2, \alpha_3] = (0.9060, 0.8259, 1.4327)$, and

$$\mathbf{M} = \begin{bmatrix} 0.67 & 0.21 & 0.14 \\ 0.33 & 0.71 & 0.08 \\ 0.00 & 0.08 & 0.78 \end{bmatrix} \begin{bmatrix} 0.9060 & 0 & 0 \\ 0 & 0.8259 & 0 \\ 0 & 0 & 1.4327 \end{bmatrix} = \begin{bmatrix} 0.6070 & 0.1734 & 0.2006 \\ 0.2990 & 0.5864 & 0.1146 \\ 0 & 0.0661 & 1.1175 \end{bmatrix}$$

Analog NTSC Color Standard

- First, define the "luminance" component of the gamma-corrected RGB values

$$Y = 0.326\tilde{r} + 0.578\tilde{g} + 0.096\tilde{b}$$

- Then, define the YPrPb coordinates system as $\begin{bmatrix} Y \\ Pb \\ Pr \end{bmatrix} = \begin{bmatrix} Y \\ \tilde{b} - Y \\ \tilde{r} - Y \end{bmatrix}$

- Then, YUV coordinates are defined as $\begin{bmatrix} Y \\ U \\ V \end{bmatrix} = \begin{bmatrix} Y \\ Pb/2.03 \\ Pr/1.14 \end{bmatrix}$

- Then, YIQ is a 33^o rotation of the UV color space

$$\begin{bmatrix} \tilde{Y} \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\sin 33^o & \cos 33^o \\ 0 & \cos 33^o & \sin 33^o \end{bmatrix} \begin{bmatrix} Y \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.274 & -0.322 \\ 0.211 & -0.523 & 0.312 \end{bmatrix} \begin{bmatrix} \tilde{r} \\ \tilde{g} \\ \tilde{b} \end{bmatrix}$$

- Comments:
  - Technically, YPbPr, YUV and YIQ assume NTSC 601 primaries with C white point.
  - Same transformations may be used with other white point and color primaries.
  - All transformations are performed on gamma corrected RGB.
  - Nominal bandwidth for Y, I, and Q channels are 4.2MHz, 1.5MHz, and 0.6MHz.

Digital NTSC Color Standard

- Assuming that $(\tilde{r}, \tilde{g}, \tilde{b})$ are scaled from 0 to 1, then...
  - First, define the "luminance" component of the gamma-corrected RGB components

$$Y = 0.326\tilde{r} + 0.578\tilde{g} + 0.096\tilde{b}$$

  - Values of YCrCb are then given by

$$\begin{bmatrix} Y_d \\ c_b \\ c_r \end{bmatrix} = \begin{bmatrix} 219Y + 16 \\ \frac{112(\tilde{b}-Y)}{0.886} + 128 \\ \frac{112(\tilde{r}-Y)}{0.701} + 128 \end{bmatrix}$$

- Complete transformation assuming $(\tilde{r}, \tilde{g}, \tilde{b})$ range from 0 to 255

$$\begin{bmatrix} Y_d \\ c_b \\ c_r \end{bmatrix} = \begin{bmatrix} 16 \\ 128 \\ 128 \end{bmatrix} + \frac{1}{255} \begin{bmatrix} 65.738 & 129.057 & 25.064 \\ -37.945 & -74.494 & 112.439 \\ 112.439 & -94.154 & -18.285 \end{bmatrix} \begin{bmatrix} \tilde{r} \\ \tilde{g} \\ \tilde{b} \end{bmatrix}$$

- Again, transformations may be used with other color primaries and white points.

Opponent Color Spaces

- Perception of color is usually not best represented in RGB.
- A better model of HVS is the so-call opponent color model
- Opponent color space is has three components:
  - O_1 is luminance component
  - O_2 is the red-green channel (G-R)
  - O_3 is the blue-yellow channel (B-Y)
- Comments:
  - People don't perceive redish-greens, or bluish-yellows.
  - As we discussed, O_1 is has a bandpass CSF.
  - O_2 and O_3 have low pass CSF's with lower frequency cut-off.
- Practical consequences:
  - Analog video has less bandwidth in I and Q channels.
  - Chromanance components are typically subsampled 2-to-1 in image compression applications.
  - Black text on white paper is easy to read. (couples to O_1)
  - Yellow text on white paper is difficult to read. (couples to O_3)

Opponent Color Space of Wandell

- First define the LMS color system which is approximately given by

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} 0.2430 & 0.8560 & -0.0440 \\ -0.3910 & 1.1650 & 0.0870 \\ 0.0100 & -0.0080 & 0.5630 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- The opponent color space transform is then[1]

$$\begin{bmatrix} O_1 \\ O_2 \\ O_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -0.59 & 0.80 & -0.12 \\ -0.34 & -0.11 & 0.93 \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

- Putting these transforms together, we get the following

$$\begin{bmatrix} O_1 \\ O_2 \\ O_3 \end{bmatrix} = \begin{bmatrix} 0.2706 & 0.7708 & 0.0136 \\ -0.1484 & 0.2064 & -0.0595 \\ -0.0904 & -0.3510 & 0.5134 \end{bmatrix} \begin{bmatrix} r_{CIE} \\ g_{CIE} \\ b_{CIE} \end{bmatrix}$$

- Comments:
  - O_1 is luminance component
  - O_2 is referred to as the red-green channel (G-R)
  - O_3 is referred to as the blue-yellow channel (B-Y)
  - Also is the work of Mullen '85[2] and associated color transforms.[3]

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[1] B. A. Wandell, *Foundations of Vision*, Sinauer Associates, Inc., Sunderland MA, 1995.

[2] K. T. Mullen, "The contrast sensitivity of human color vision to red-green and blue-yellow chromatic gratings," *J. Physiol.*, vol. 359, pp. 381-400, 1985.

[3] B. W. Kolpatzik and C. A. Bouman, "Optimized Error Diffusion for Image Display," *Journal of Electronic Imaging*, vol. 1, no. 3, pp. 277-292, July 1992.

Paradox?

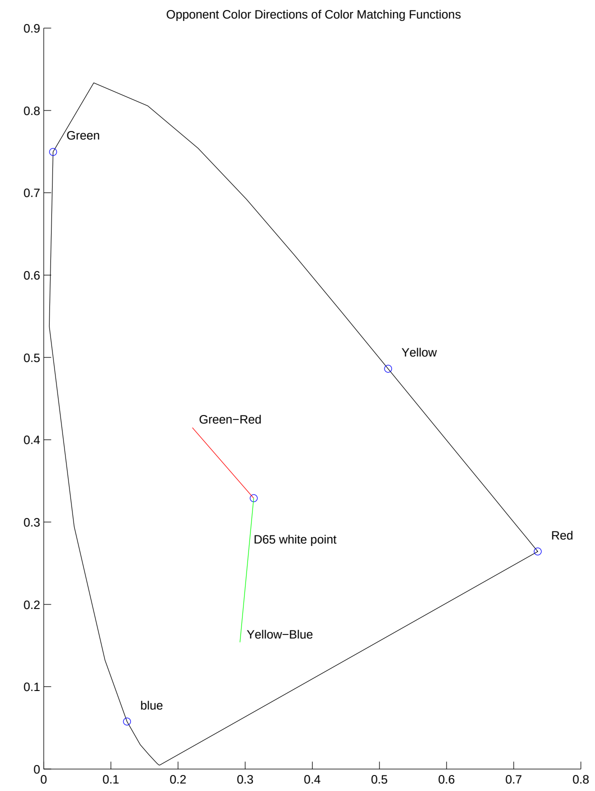
- Why is blue text on yellow paper is easy to read??
- Shouldn't this be hard to read since it stimulates the yellow-blue color channel?

Better Understanding Opponent Color Spaces

- The XYZ to opponent color transformation is:

$$\begin{bmatrix} O_1 \\ O_2 \\ O_3 \end{bmatrix} = \begin{bmatrix} 0.2430 & 0.8560 & -0.0440 \\ -0.4574 & 0.4279 & 0.0280 \\ -0.0303 & -0.4266 & 0.5290 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} v_y \\ v_{gr} \\ v_{yb} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

where v_y, v_{gr}, and v_{yb} are row vectors in the XYZ color space.



- These row vectors are sometimes refered to as *contravariant vectors*.
- They are not orthogonal!

$$\begin{bmatrix} v_y \\ v_{gr} \\ v_{yb} \end{bmatrix} \left[v_y^t \, v_{gr}^t \, v_{yb}^t \right] \neq \text{identity matrix}$$

- They are not the basis functions for the opponent color space.

Basis Vectors for Opponent Color Spaces

- The transformation from opponent color space to XYZ is:

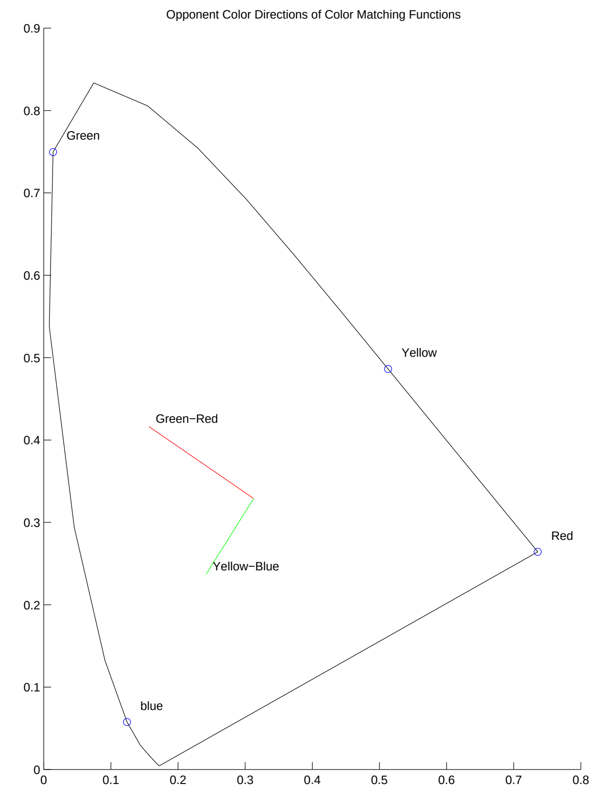
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.9341 & -1.7013 & 0.1677 \\ 0.9450 & 0.4986 & 0.0522 \\ 0.8157 & 0.3047 & 1.9422 \end{bmatrix} \begin{bmatrix} O_1 \\ O_2 \\ O_3 \end{bmatrix} = [c_y \, c_{gr} \, c_{yb}] \begin{bmatrix} O_1 \\ O_2 \\ O_3 \end{bmatrix}$$

where c_y, c_{gr}, and c_{yb} are column vectors.

- These column vectors are sometimes refered to as *co-variant vectors*.
- They are orthogonal to the row vectors v_y, v_{gr}, and v_{yb}.

$$\begin{bmatrix} v_y \\ v_{gr} \\ v_{yb} \end{bmatrix} [c_y \, c_{gr} \, c_{yb}] = \text{identity matrix}$$

- They are the basis functions for the opponent color space.



Solution to Paradox?

- Why is blue text on yellow paper is easy to read??
- Solution:
  - The blue-yellow combination generates the input v_{yb}.
  - This input vector stimulates all three opponent channels because it is not orthononal to c_y, c_{gr}, and c_{yb}.
  - In particlar, it strongly stimulates c_y because it is **not** iso-luminant.

Perceptually Uniform Color Spaces

- Problem: Small changes in XYZ may result in small or large perceptual changes.
- Solution: Formulate a perceptually uniform color space.
  - Select (X_0, Y_0, Z_0) to be the white point or illuminant.
  - Then compute (approximate formula)

$$\begin{aligned} L &= 100(Y/Y_0)^{1/3} \\ a &= 500\left[(X/X_0)^{1/3} - (Y/Y_0)^{1/3}\right] \\ b &= 200\left[(Y/Y_0)^{1/3} - (Z/Z_0)^{1/3}\right] \end{aligned}$$

  - Color errors can then be measured as:

$$\Delta E = \sqrt{(\Delta L)^2 + (\Delta a)^2 + (\Delta b)^2}$$

- **Danger:**
  - Lab color space is designed for low spatial frequencies.
  - Direct application to images works poorly.
  - Lab formulation ignores different form of CSF for luminance and chrominance spaces.

Color Image Quality Metrics

- A number of attempts have been made to combine CSF modeling with Lab color space:
  - Kolpatzik and Bouman '95 (YCxCz/Lab color metric)[4]
  - Zhang and Wandell '97 (S-CIELAB color metric)[5]
- Objective:
  - Combine models of spatial frequency response and color space nonuniformities.
  - Particularly important form modeling halftone quality due to high frequency content.

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[4] B. W. Kolpatzik and C. A. Bouman, "Optimized Universal Color Palette Design for Error Diffusion," *Journal of Electronic Imaging,* vol. 4, no. 2, pp. 131-143, April 1995.

[5] X. Zhang and B. A. Wandell, "A spatial extension of CIELAB for digital color image reproduction," *Society for Information Display Journal,* 1997.

Image Quality Metric Using YCxCz/Lab

- YCxCz color space is defined as:

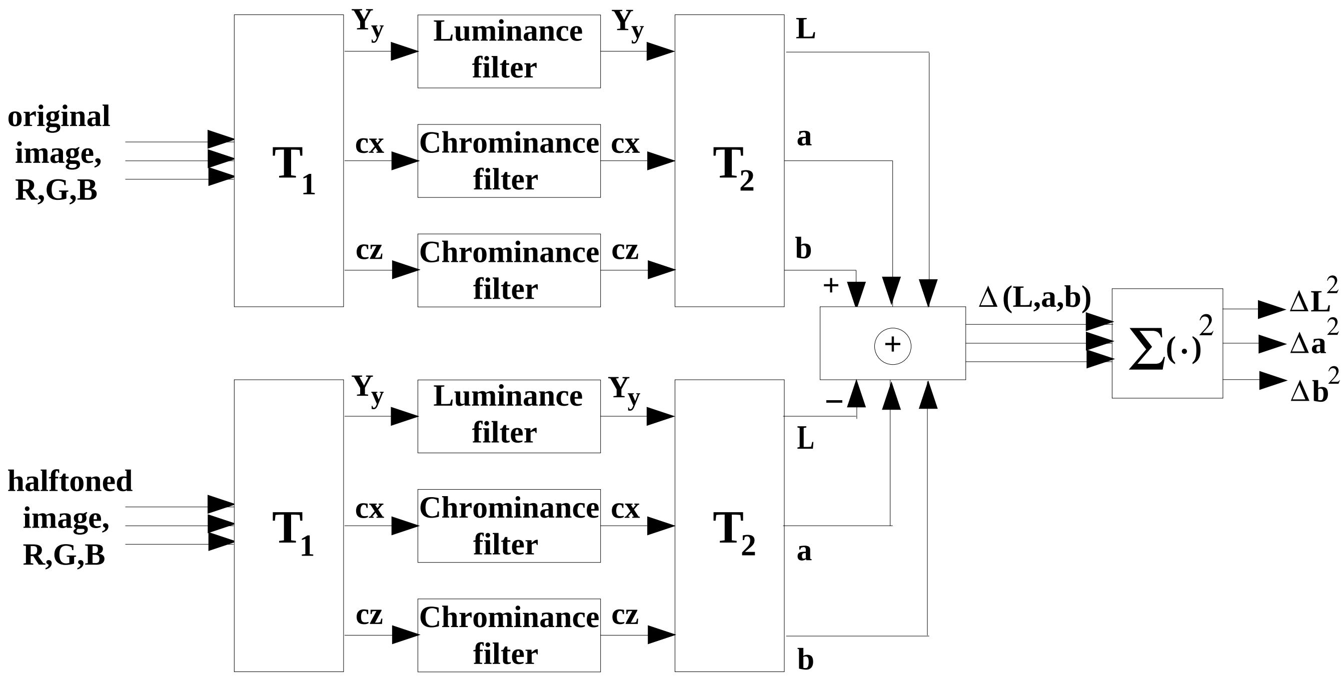
$$
\begin{aligned}
Y_y &= 116(Y/Y_0) \\
c_x &= 500\left[(X/X_0) - (Y/Y_0)\right] \\
c_z &= 200\left[(Y/Y_0) - (Z/Z_0)\right]
\end{aligned}
$$

- Apply different filters for luminance and chromanance where f is in units of cycles/degree.

$$
\begin{aligned}
W_y(f) &= \begin{cases} \exp\{-0.4385(f - 2.2610)\} & f \geq 2.2610 \\ 1 & f < 2.2610 \end{cases} \\
W_{cx}(f) = W_{cz}(f) &= \begin{cases} \exp\{-0.1761(f - 0.2048)\} & f \geq 0.2048 \\ 1 & f < 0.2048 \end{cases}
\end{aligned}
$$

- Then transform filtered YCxCz image components to Lab and compute ΔE.

Flow Diagram for YCxCz/Lab Image Quality Metric



- Low pass filters are applied in linear domain $\Rightarrow$ more accurate for color matching of halftones.
- Nonlinear Lab transformation accounts for perceptual nonuniformities of color space.