EE 637 Final Exam

May 9, Spring 1997

Name: ________________________________

Instructions: Two hour exam, with no books, notes or calculators.

Problem 1.(20pt)

An imaging system has an impulse respose of

$$g(m,n) = \delta(m,n) + \alpha h(m,n)$$

where $0 < \alpha << 1$, and $h(m,n)$ is defined by

$$h(m,n) = \begin{cases} 1/9 & |m| \leq 1 \text{ and } |n| \leq 1 \\ 0 & \text{otherwise} \end{cases} .$$

a) Compute the frequency response of the imaging system, $G(e^{j\mu}, e^{j\nu})$, and explain the effect of this response on images.

b) Compute a filter $F(e^{j\mu}, e^{j\nu})$ that approximately corrects for the distortion of the imaging system by using the relationship that

$$\frac{1}{1+a} \approx 1 - a$$

c) Compute the impulse response of the correcting filter, $f(m,n)$, and explain the effect of the filter on images.

Problem 2.(20pt)

Let $X_{m,n}$ be i.i.d. samples from the distribution $N(0,1)$. Let $Y_{m,n}$ be a derived from $X_{m,n}$ using the difference equation

$$Y_{m,n} = X_{m,n} + aY_{m-1,n} + aY_{m,n-1} - a^2 Y_{m-1,n-1} \tag{1}$$

where $0 < a < 1$.

a) Compute, $S_y(e^{j\mu,j\nu})$, the power spectrum of $Y_{m,n}$.

b) Is equation (1) a stable system? Prove or disprove your result.

Let $Z_{m,n}$ be derived from $Y_{m,n}$ by applying a filter $h_{m,n}$.

c) Determine **all** filters $h_{m,n}$ so that the samples $Z_{m,n}$ are i.i.d.

Problem 3.(20pt)

A subject is presented with a achromatic visual stimulus (i.e. an image) at a distance d and with a luminance of

$$L(x, y) = A + B\sin(2\pi f(x + y))$$

where x an y are horizontal and vertical coordinates of the stimulus respectively.

a) Compute the number of cycles per degree of the stimulus in terms of f and d.

Next, the value of B is adjusted so that the subject can detect the sine wave grating 50% of the time.

b) Sketch the value of the ratio B/A as a function of the value A. Explain the shape of your plot. Assume the f is choosen so that the stimulus is at approximately 5 cycles per degree.

c) Sketch the value of the ratio B/A as a function of the value f. Explain the shape of your plot. Assume the A is choosen to be in the photopic range of the visual system.

Problem 4.(20pt)

Let $X_{m,n}$ be a discrete valued random image with i.i.d pixels that take on values in the set $\{0, 1, \cdots, 6\}$. Furthermore, let the marginal distribution of $X_{m,n}$ be given by

x	0	1	2	3	4	5	6
$P\{X_{m,n} = x\}$	0.40	0.30	0.08	0.07	0.06	0.05	0.04

Design a Huffman code for the symbols of this image and do the following:

a) Show the binary tree for your code. Label the probability of each node and the binary value for each branch.

b) List the binary code for each symbol.

c) Compute the expected code length.

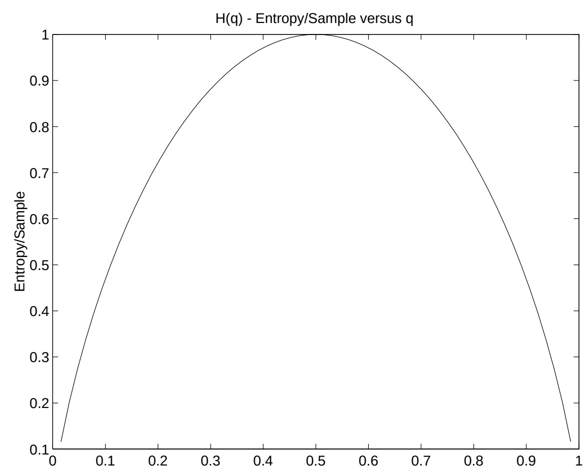
d) Give a strategy for designing a code with reduced bit rate. Be specific, and explain how much the bit rate may be reduced.

Problem 5.(20pt)

Let $X_{m,n}$ be a random binary valued image which you need to compress using a lossless predictive coder. In order to design the coder, you first measure statistics from sample images, $x_{m,n}$, to form a histogram $h_{i,j,k}$. The histogram is given by

$x_{m,n} = i$	$x_{m-1,n} = j$	$x_{m,n-1} = k$	$h_{i,j,k}$
0	0	0	30
0	0	1	6
0	1	0	2
0	1	1	2
1	0	0	2
1	0	1	2
1	1	0	6
1	1	1	14

The following figure is a plot of the function $H(q) = -q\log_2(q) - (1-q)\log_2(1-q)$



a) Compute an estimate for for the marginal distribution of $X_{m,n}$, $p_i = P\{X_{m,n} = i\}$.

b) Given the result of part a) and the entropy plot, compute the per sample entropy of $X_{m,n}$ under the assumption that pixels are i.i.d.

c) Compute an estimate for for the joint distribution of $(X_{m-1,n}, X_{m,n-1})$,

$$g_{jk} = P\{X_{m-1,n} = j, X_{m,n-1} = k\} .$$

d) Compute the predictor $\hat{X}(j,k)$ which minimizes the probability of error given $X_{m-1,n} = j, X_{m,n-1} = k$.

e) Define the image of prediction errors $Z_{m,n}$ so that

$$Z_{m,n} = \begin{cases} 0 & X_{m,n} = \hat{X}(j,k) \\ 1 & X_{m,n} \neq \hat{X}(j,k) \end{cases}$$

Given the result of part d) and the entropy plot, compute the per sample entropy of $Z_(m,n)$ under the assumption that its pixels are i.i.d.