M-estimators

Chose some function $\rho(\theta)$, such that

$$\rho(\Delta) = \rho(-\Delta)$$

and

$$\frac{d}{d\theta}\rho(\Delta) = \rho'(\Delta) \begin{cases} \text{best if} \\ \text{exists } \forall\ \Delta \in \mathbb{R} \\ \text{is continuous } \Delta \in \mathbb{R} \end{cases}$$

- $\rho'(\theta)$ is known as the "influence" function

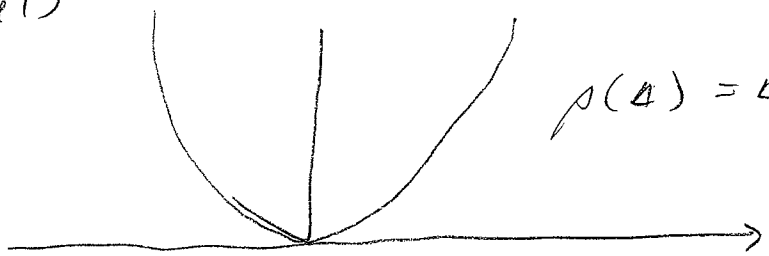
$$Y(m) = \sum_{k \in W(m)} \rho(x(k) - \theta)$$

$$\frac{d}{d\theta} \sum_{k \in W(m)} \rho(x(k) - \theta) =$$

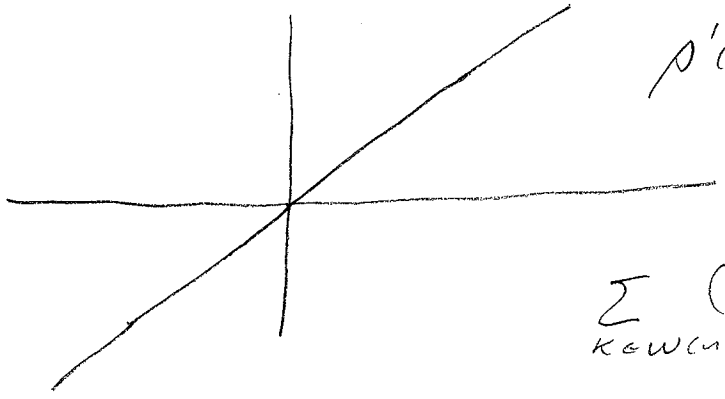
$$= \sum_{k \in W(m)} \rho'(x(k) - \theta) = 0$$

↑ like a force balance equation

Example 1)



$\rho(\Delta) = \Delta^2$ $\quad \Delta = x(k) - \theta$



$\rho'(\Delta) = \Delta$

$$\sum_{k \in W(m)} (x(k) - \theta) = 0$$

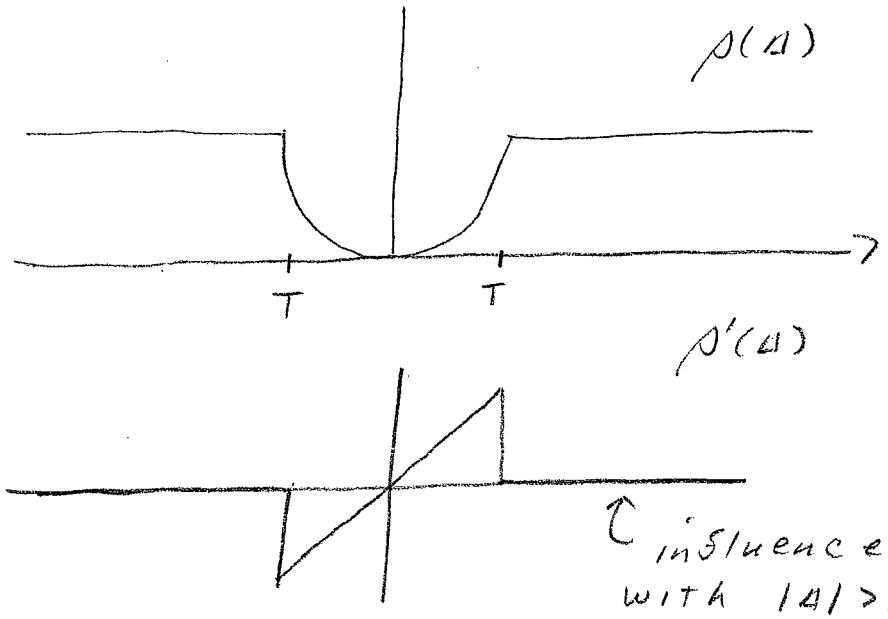
$$\theta = \frac{1}{p} \sum_{k \in W(m)} x(k)$$

↑ mean

- outliers pixels have large effect.

Example 2)

$$\rho(\Delta) = \begin{cases} \Delta^2 & |\Delta| < T \\ T^2 & |\Delta| > T \end{cases}$$



↑ influence of pixels with $|\Delta| > T$ is zero!

- Problem: $\rho(\Delta)$ is not convex ⇒ optimization is difficult.

Example 2)

$$\rho(\Delta) = \begin{cases} \Delta^2 & |\Delta| < T \\ 2T|\Delta| - T^2 & |\Delta| \geq T \end{cases}$$

- Known as "Huber function"