TECHNICAL NOTES

Optimal short scan convolution reconstruction for fanbeam CT

Dennis L. Parker

Department of Radiation Oncology, University of California at San Francisco, San Francisco, California 94143

(Received 22 June 1981; accepted for publication 23 October 1981)

The problem of using a divergent fan beam convolution reconstruction algorithm in conjunction with a minimal complete (180° plus the fan angle) data set is reviewed. It is shown that by proper weighting of the initial data set, image quality essentially equivalent to the quality of reconstructions from 360° data sets is obtained. The constraints on the weights are that the sum of the two weights corresponding to the same line-integral must equal one, in regions of no data the weights must equal zero, and the weights themselves as well as the gradient of the weights must be continuous over the full 360°. After weighting the initial data with weights that satisfy these constraints, image reconstruction can be conveniently achieved by using the standard (hardwired if available) convolver and backprojector of the specific scanner.

I. INTRODUCTION

In the development of fast, special purpose CT scanners, a reduction in scanning time is also obtained by collecting only the minimum number of projections (views) required for image reconstruction. It is known, for example, that a scanner with a parallel beam geometry (such as a translate/rotate scanner) requires projections for 180°.[1] It has also been shown that data collected from fanbeam geometries can be rebinned or reorganized into parallel ray data sets for reconstruction with parallel geometry algorithms.[2] It is easy to show that a complete parallel ray equivalent data set can be obtained from a set of divergent ray projections taken over 180° plus the angle of the divergent fan beam. It is logical to refer to this set of divergent projections which covers 180° plus the fan angle as the "minimal complete data set". It is the minimum set of equally spaced projection measurements which can be used in conventional convolution type reconstruction algorithms. In this note we show that with proper weighting of the input data set it is possible to reconstruct the minimal complete data set using conventional divergent convolution reconstruction algorithms (i.e., with no rebinning into parallel ray data sets). It is therefore possible to utilize conventional array processors and standard hardwired backprojectors in performing the reconstruction of data from short scans.

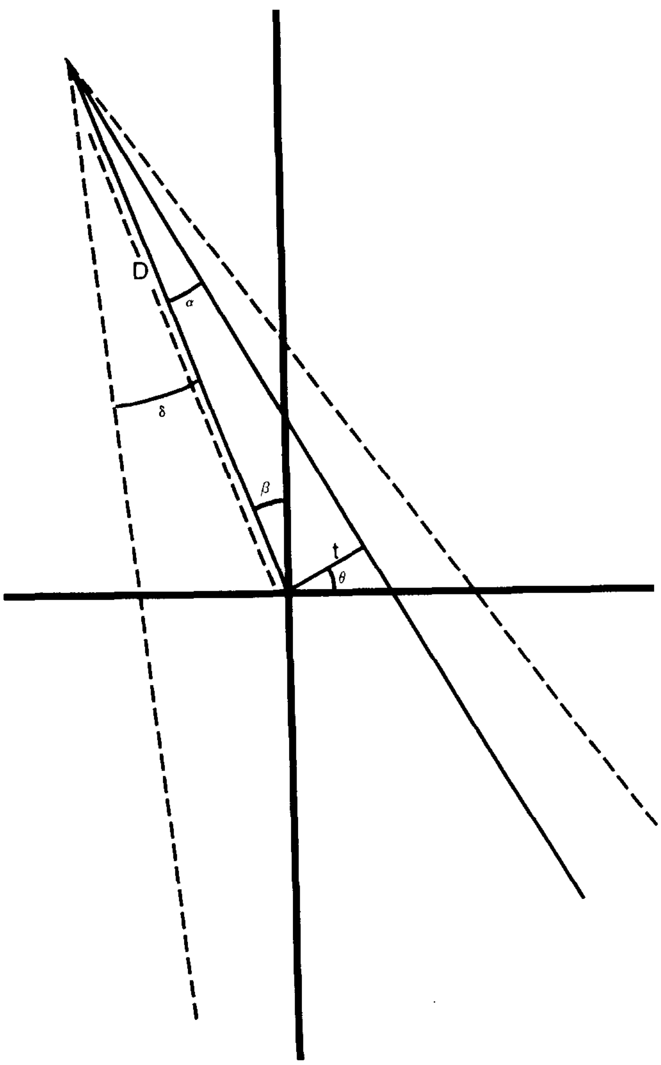


FIG. 1. Scanning geometry utilized in conventional fanbeam scanners.

In a paper by Naparstek,[3] several methods of short scan fan beam reconstruction techniques were derived and assessed. However, those algorithms which generated images of reasonable quality also required more projections than a minimal complete set. The more successful algorithms utilized a smooth weighting of the extra projections in a manner similar to that used in standard CT overscanning for artifact reduction.[4] In this note, we show that by a simple modification of one of the techniques of Naparstek, good quality images can be generated from a minimal complete short scan data set.

II. ALGORITHM DEVELOPMENT

Figure 1 illustrates the geometry of fan beam data collection where β is the angle of rotation of the source to axis

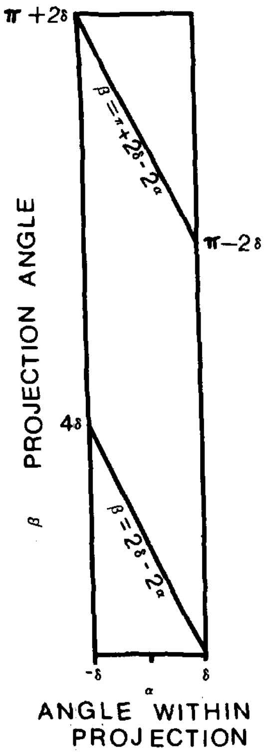


FIG. 2. Outline of sampling region as a function of α (the horizontal coordinate) and β (the vertical coordinate). The upper triangle is geometrically the same (i.e., it samples the same set of line integrals) as the lower triangle.

segment and α is the relative angular position of an individual line integral from the source to a detector element. If $g(\alpha,\beta)$ is the projection measurement (line integral) along the segment specified by (α,β) and if $p(t,\vartheta)$ is the corresponding parallel ray projection measurement where (t,ϑ) are defined in the figure, it is evident that

$$p(t,\vartheta) = g(\alpha,\beta), \tag{1}$$

when

$$\vartheta = \beta + \alpha$$

and

$$t = D \sin \alpha.$$

The symmetry over rotation requires that

$$p(t,\vartheta) = p(-t,\vartheta + \pi), \tag{2}$$

which is equivalent to

$$g(\alpha,\beta) = g(-\alpha,\pi + \beta + 2\alpha). \tag{3}$$

Thus, if data is collected in the rectangle,

$$-\delta < \alpha < \delta,$$

$$0 < \beta < \pi + 2\delta, \tag{4}$$

where 2δ is the fan angle defined in Fig. 1, then the data in the triangle,

$$0 < \beta < 2\delta - 2\alpha,$$

will be positionally redundant with the data in the region:

$$\pi - 2\alpha < \beta < \pi + 2\delta.$$

This is illustrated in Fig. 2. The set of line integrals specified by the upper triangle is resampled in the lower triangle as required by Eq. (3). In contrast, the set of line integrals specified by the parallelogram region bounded by the two triangles is only sampled once.

Because only part of the region in the (α,β) diagram is doubly scanned, it is expected that direct application of any direct divergent fanbeam reconstruction algorithm which itself is based on total double scanning (360°) would yield a distorted image. That this is true is illustrated using the simulated data of Fig. 3. The mathematical phantom of Fig. 3(a) yields the divergent sinogram of Fig. 3(b) for a third generation (rotate–rotate) scanner with 121 detectors at 1/3-degree angular increments for a fan angle of 40° and 360 views taken at one-degree projection increments. The data has been convolved with a simple divergent algorithm kernel[5]:

$$q_i = \frac{-1}{\pi^2 \sin^2(i\Delta\alpha)},\ i,\text{odd},$$

$$q_i = 0,\ i,\text{even}\ i \neq 0,$$

$$q_0 = \frac{1}{4\Delta\alpha^2}.\ i = 0. \tag{5}$$

When the entire (360°) data set is reconstructed using a standard backprojection algorithm, the image of Fig. 3(c) is obtained. If, however, the case of a minimal complete data set is imitated by considering only the first 220 projections in the standard backprojection algorithm, the image of Fig. 3(d) is obtained. The variation in intensity over the image is surprisingly smooth and the image itself is very recognizable.

It might be expected that the smooth variation could be removed by setting the data from one of the redundant triangles to zero. It was shown by Naparstek[3] that the streak artifacts which resulted from the zeroed boundary of the triangle were sufficient to completely mask the image. Of the other algorithms proposed by Naparstek, the most successful ones required additional data (beyond the minimal complete short scan data set). This additional data was then used to feather smoothly to zero the abrupt discontinuity at the parallelogram boundary. In addition to requiring data beyond that of a minimal complete data set, the algorithms also generally required a more complicated convolution kernel.

The logical extension, which was not formally proposed or attempted by Naparstek, is to weight the data of the minimal complete data set in such a manner that the discontinuity is as uniformly distributed as possible. The constraints on the set of weights can be developed as follows: The weighted data set is defined as $g'(\alpha,\beta)$ and is given by

$$g'(\alpha,\beta) = \omega(\alpha,\beta) g(\alpha,\beta), \tag{6}$$

where

$$\omega(\alpha,\beta) + \omega(-\alpha,\pi + \beta + 2\alpha) = 1. \tag{7}$$

The minimization of the discontinuities at the various borders of the triangles is accomplished by the requirements

$$\omega(\alpha,0) = \omega(\alpha,\pi + 2\delta) = 0, \tag{8}$$

$$\omega(\alpha,2\delta - 2\alpha) = \omega(\alpha,\pi - 2\alpha) = 1, \tag{9}$$

$$\left.\frac{\partial\omega(\alpha,\beta)}{\partial\beta}\right|_{\beta = 2\delta - 2\alpha} = \left.\frac{\partial\omega(\alpha,\beta)}{\partial\beta}\right|_{\beta = \pi - 2\alpha} = 0, \tag{10}$$

$$\left.\frac{\partial\omega(\alpha,\beta)}{\partial\alpha}\right|_{\beta = 2\delta - 2\alpha} = \left.\frac{\partial\omega(\alpha,\beta)}{\partial\alpha}\right|_{\beta = \pi - 2\alpha} = 0. \tag{11}$$

One set of weights which satisfies all of the above constraints is given by

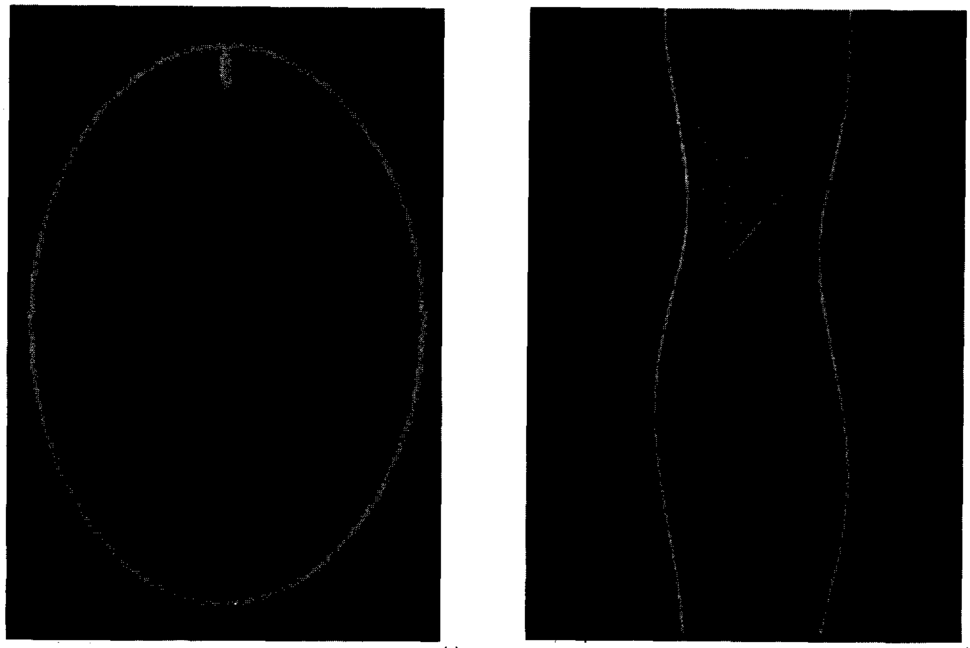
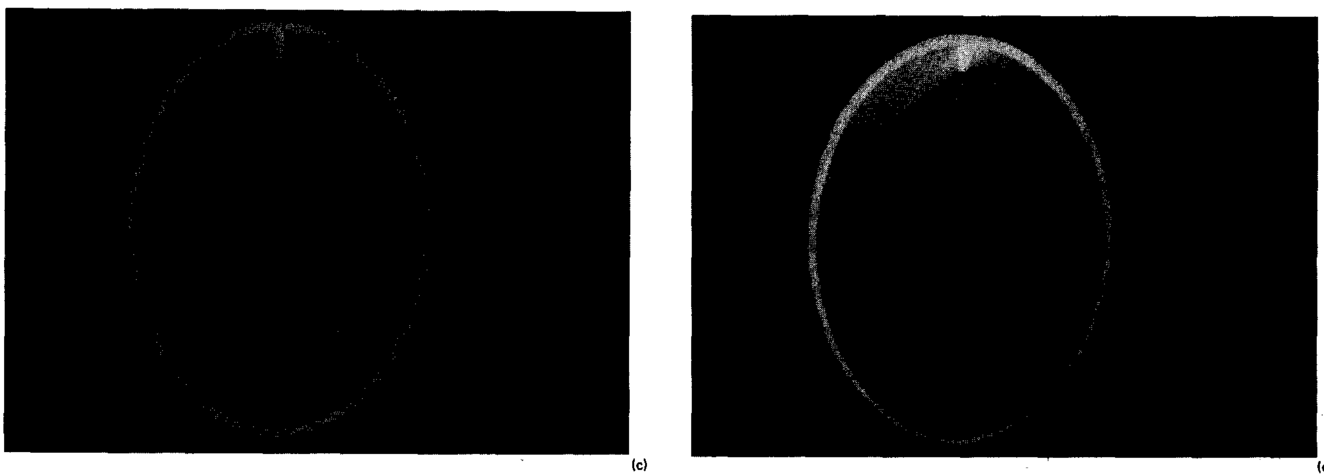


FIG. 3. (a) The mathematical phantom used in the simulations. See text for description. (b) The set of projection measurements in the same format as Fig. 2 after convolving with the reconstruction kernel. See text for geometry of simulated scanner. Because line integrals of negligible width were calculated, the artifacts evident are due to aliasing. (c) The reconstruction of the data of Fig. 3(b) using a standard divergent backprojection algorithm and projection data over 360°. (d) The reconstruction of the data of Fig. 3(b) using the same algorithm but with projection data over only 220°.



$$\omega(\alpha,\beta) = \sin^2\left(\frac{\pi}{4}\frac{\beta}{\delta-\alpha}\right),\ 0\leqslant\beta\leqslant 2\delta - 2\alpha\ ,$$

$$\omega(\alpha,\beta) = 1,\ 2\delta - 2\alpha\leqslant\beta\leqslant\pi - 2\alpha\ ,$$

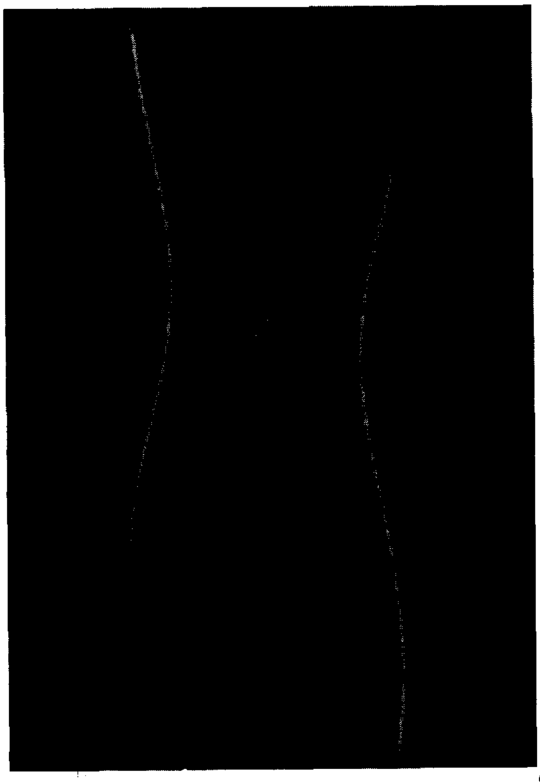
$$\omega(\alpha,\beta) = \sin^2\left(\frac{\pi}{4}\frac{\pi+2\delta-\beta}{\alpha}\right).\ \pi - 2\alpha\leqslant\beta\leqslant\pi + 2\delta\ . \quad (12)$$

III. RESULTS

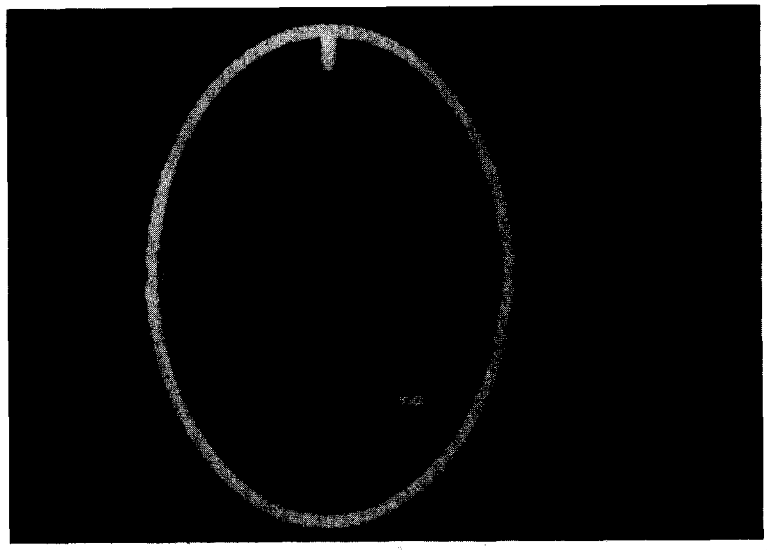
In Fig. 4(a) we show the sinogram from Fig. 3(b) weighted by the functions of Eq. (12). Figure 4(b) is the reconstruction of the data from 4(a) using a standard divergent fanbeam backprojection algorithm. [This is the same algorithm as used in Figs. 3(c) and (d).] It is evident in comparing Figs. 3(c) and 4(b) that there is no appreciable difference in image quality between the full 360° reconstruction and the 220°, weighted reconstruction.

IV. CONCLUSION

The technique which has been presented has demonstrated the ability to reconstruct images from a minimal complete data set (180° plus the fan angle) with no evident loss in image quality over that obtained from reconstruction of the full 360° scan data. The technique requires the smooth weighting of data in double-scanned regions while the weights and their derivatives are required to be continuous at the boundaries. After the weighting is performed, standard divergent convolution/backprojection algorithms are used to reconstruct the image. It is therefore possible to maintain the hardware-related reconstruction speed of standard convolvers and backprojectors while significantly reducing the required number of scanned projections, thereby decreasing the associated scan time. In this manner artifacts related to the duration of the scan, such as motion artifacts or electronic drifts in the data acquisition system, etc., can be minimized.



(a)



(b)

FIG. 4. (a) The projection data over 220° where the input data has been weighted by the weights given in the text and then convolved with the same algorithm as that of Fig. 3(b). (b) The reconstruction of the data of Fig. 4(a) using the same algorithm as Figs. 3(c) and (d).

ACKNOWLEDGMENTS

The author wishes to acknowledge Dr. Douglas Boyd for helpful suggestions. The author is supported by a grant from Toshiba Corp.

[1]L. A. Shepp and B. F. Logan, IEEE Trans. Nucl. Sci. **NS-21**, 21 (1974).
[2]P. Dreike and D. P. Boyd, Comput. Graph. Image Proc. **5**, 459 (1976).
[3]A. Naparstek, IEEE Trans. Nucl. Sci. **NS-27**, 1112 (1980).
[4]D. L. Parker, V. Smith, and J. H. Stanley, Med. Phys. **8**, 706 (1981).
[5]G. T. Herman, A. V. Lakshminarayanan, and A. Naparstek, Comput. Biol. Med. **6**, 259 (1976).