What is Color?

- Color is a human perception (a percept).
- Color is not a physical property...
- But, it is related the the light spectrum of a stimulus.

Can We Measure the Percept of Color?

- Semantic names - red, green, blue, orange, yellow, etc.
- These color semantics are largely culturally invariant, but not precisely.
- Currently, there is no accurate model for predicting perceived color from the light spectrum of a stimulus.
- Currently, noone has an accurate model for predicting the percept of color.

Can We Tell if Two Colors are the Same?

- Two colors are the same if they match at *all* spectral wavelengths.
- However, we will see that two colors are also the same if they match on a 3 dimensional subspace.
- The values on this three dimensional subspace are called *tristimulus* values.
- Two colors that match are called *metamers*.

Matching a Color Patch

- Experimental set up:
  - Form a reference color patch with a known spectral distribution.

$$\text{Reference Color} \Rightarrow I(\lambda)$$

  - Form a second adjustable color patch by adding light with three different spectral distributions.

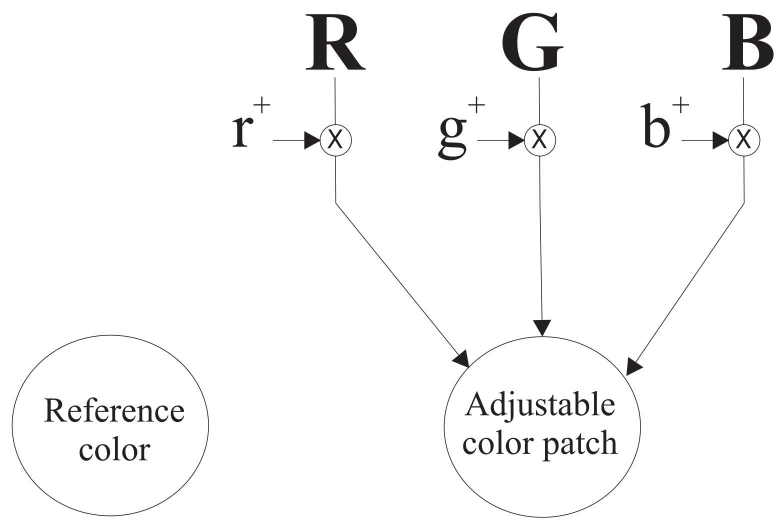
$$\begin{aligned} \text{Red} &\Rightarrow I_r(\lambda) = \mathbf{R} \\ \text{Green} &\Rightarrow I_g(\lambda) = \mathbf{G} \\ \text{Blue} &\Rightarrow I_b(\lambda) = \mathbf{B} \end{aligned}$$

  - Control the amplitude of each component with three individual positive constants r^+, g^+, and b^+.
  - The total spectral content of the adjustable patch is then

$$r^+ I_r(\lambda) + g^+ I_g(\lambda) + b^+ I_b(\lambda) .$$

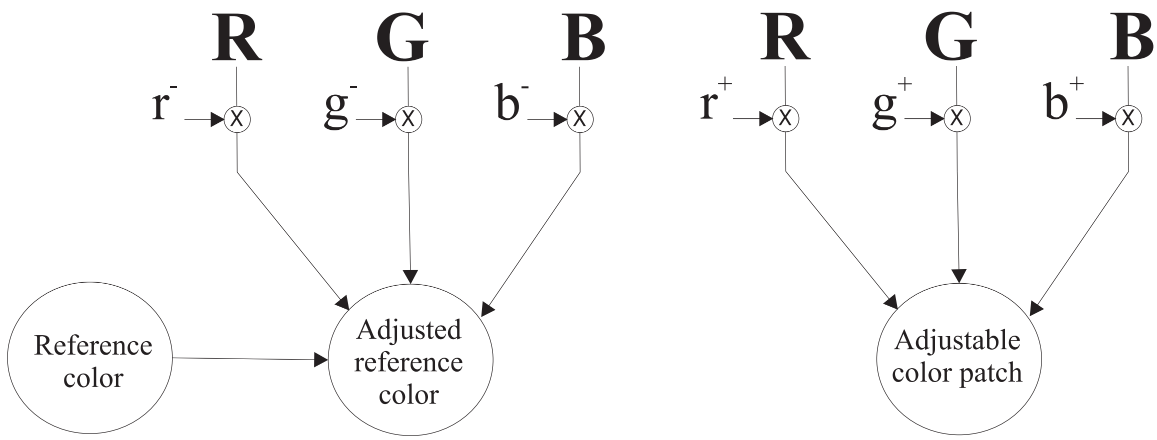
- Choose (r^+, g^+, b^+) to match the two color patches.

Simple Color Matching with Primaries



- Choose (r^+, g^+, b^+) to match the two color patches.
- The values of (r, g, b) must be positive!
- Definitions:
  - $\mathbf{R}$, $\mathbf{G}$, and $\mathbf{B}$ are known as color primaries.
  - r^+, g^+, and b^+ are known as tristimulus values.
- Problem:
  - Some colors can not be matched, because they are too "saturated".
  - These colors result in values of r^+, g^+, or b^+ which are 0.
  - How can we generate negative values for r^+, g^+, or b^+?

Improved Color Matching with Primaries



- Add color primaries to reference color!
- This is equivalent to subtracting them from adjustable patch.
- Equivalent tristimulus values are:

$$r = r^{+} - r^{-}$$

$$g = g^{+} - g^{-}$$

$$b = b^{+} - b^{-}$$

- In this case, r, g, and b can be both positive and negative.
- All colors may be matched.

Grassman's Law

- Grassman's law: Color perception is a 3 dimensional linear space.

- Superposition:
  - Let $I_1(\lambda)$ have tristimulus values (r_1, g_1, b_1), and let $I_2(\lambda)$ have tristimulus values (r_2, g_2, b_2).
  - Then $I_3(\lambda) = I_1(\lambda) + I_2(\lambda)$ has tristimulus values of

$$(r_3, g_3, b_3) = (r_1, g_1, b_1) + (r_2, g_2, b_2)$$

- This implies that tristimulus values can be computed with a linear functional of the form

$$\begin{aligned} r &= \int_0^\infty r_0(\lambda)\, I(\lambda) d\lambda \\ g &= \int_0^\infty g_0(\lambda)\, I(\lambda) d\lambda \\ b &= \int_0^\infty b_0(\lambda)\, I(\lambda) d\lambda \end{aligned}$$

  for some functions $r_0(\lambda)$, $g_0(\lambda)$, and $b_0(\lambda)$.

- Definition: $r_0(\lambda)$, $g_0(\lambda)$, and $b_0(\lambda)$ are known as color matching functions.

Measuring Color Matching Functions

- A pure color at wavelength λ_0 is known as a line spectrum. It has spectral distribution
$$I(\lambda) = \delta(\lambda - \lambda_0) \ .$$
Pure colors can be generated using a laser or a very narrow band spectral filter.

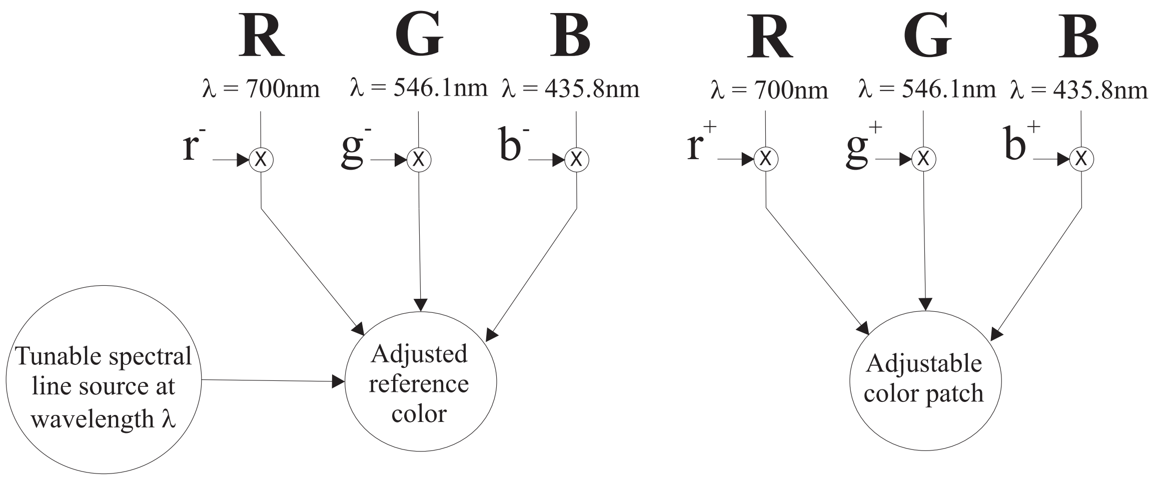
- When the reference color is such a pure color, then the tristimulus values are given by
$$\begin{aligned} r &= \int_0^\infty r_0(\lambda)\,\delta(\lambda-\lambda_0)d\lambda = r_0(\lambda_0) \\ g &= \int_0^\infty g_0(\lambda)\,\delta(\lambda-\lambda_0)d\lambda = g_0(\lambda_0) \\ b &= \int_0^\infty b_0(\lambda)\,\delta(\lambda-\lambda_0)d\lambda = b_0(\lambda_0) \end{aligned}$$

- Method for Measuring Color Matching Functions:
  - Color match to a reference color generated by a pure spectral source at wavelength λ_0.
  - Record the tristimulus values of $r_0(\lambda_0)$, $g_0(\lambda_0)$, and $b_0(\lambda_0)$ that you obtain.
  - Repeat for all values of λ_0.

CIE Standard RGB Color Matching Functions

- An organization call Commission Internationale de l'Eclairage (CIE) defined all practical standards for color measurements (colorimetery).
- CIE 1931 Standard 2^o Observer:
  - Uses color patches that subtended 2^o of visual angle.
  - $\mathbf{R}$, $\mathbf{G}$, $\mathbf{B}$ color primaries are defined by pure line spectra (delta functions in wavelength) at 700nm, 546.1nm, and 435.8nm.
  - Reference color is a spectral line at wavelength λ.
- CIE 1965 10^o Observer: A slightly different standard based on a 10^o reference color patch and a different measurement technique.

RGB Color Matching Functions for CIE Standard 2^o Observer



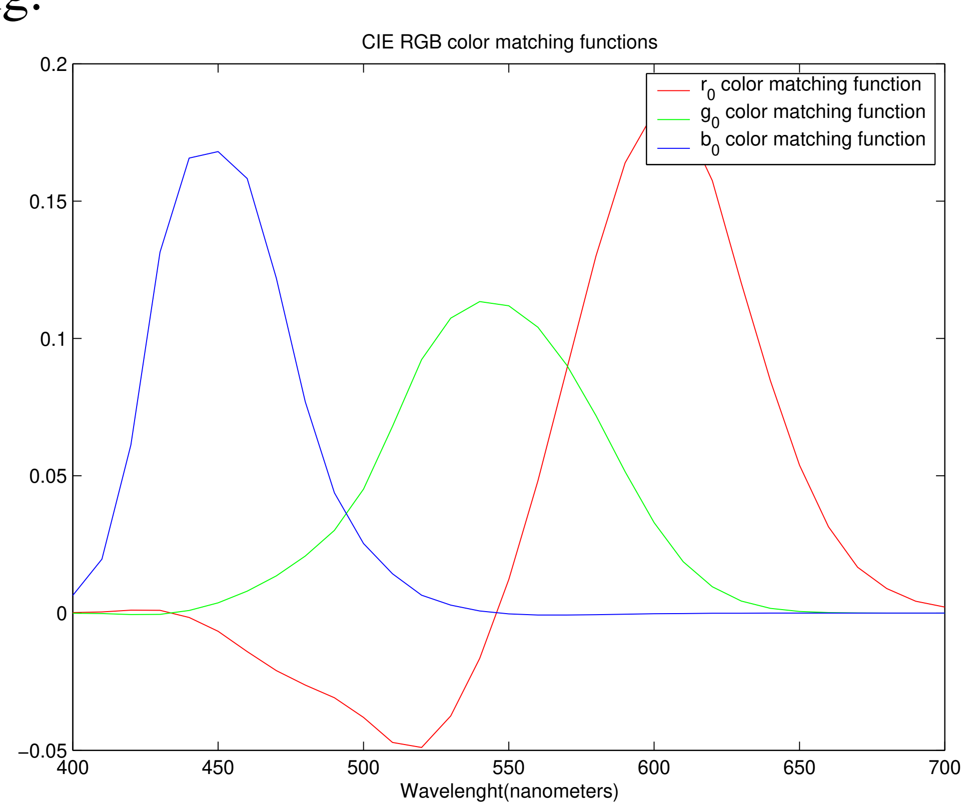
- The color matching functions are then given by

$$
\begin{aligned}
r_0(\lambda) &= r^+ - r^- \\
g_0(\lambda) &= g^+ - g^- \\
b_0(\lambda) &= b^+ - b^-
\end{aligned}
$$

where λ is the wavelength of the reference line spectrum.

RGB Color Matching Functions for CIE Standard 2^o Observer

- Plotting the values of $r_0(\lambda)$, $g_0(\lambda)$, and $b_0(\lambda)$ results in the following.



- Notice that the functions take on negative values.

Review of Colorimetry Concepts

1. $\mathbf{R}, \mathbf{G}, \mathbf{B}$ are color primaries used to generate colors.
2. (r, g, b) are tristimulus values used as weightings for the primaries.

$$\begin{aligned} \text{Color} &= r\mathbf{R} + g\mathbf{G} + b\mathbf{B} \\ &= [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} r \\ g \\ b \end{bmatrix} \end{aligned}$$

3. $(r_0(\lambda), g_0(\lambda), b_0(\lambda))$ are the color matching functions used to compute the tristimulus values.

$$\begin{aligned} r &= \int_0^\infty r_0(\lambda)\, I(\lambda) d\lambda \\ g &= \int_0^\infty g_0(\lambda)\, I(\lambda) d\lambda \\ b &= \int_0^\infty b_0(\lambda)\, I(\lambda) d\lambda \end{aligned}$$

- How are the color matching functions scaled?

Scaling of Color Matching Functions

- Color matching functions are scaled to have unit area

$$\int_0^\infty r_0(\lambda) d\lambda = 1$$

$$\int_0^\infty g_0(\lambda) d\lambda = 1$$

$$\int_0^\infty b_0(\lambda) d\lambda = 1$$

- Color "white"
  - Has approximately equal energy at all wavelengths
  - $I(\lambda) = 1$
  - White $\Leftrightarrow (r, g, b) = (1, 1, 1)$
  - Known as equal energy (EE) white
  - We will talk about this more later

Problems with CIE RGB

- Some colors generate negative values of (r, g, b).
- This results from the fact that the color matching functions $r_0(\lambda)$, $g_0(\lambda)$, $b_0(\lambda)$ can be negative.
- The color primaries corresponding to CIE RGB are very difficult to reproduce. (pure spectral lines)
- Partial solution: Define new color matching functions $x_0(\lambda)$, $y_0(\lambda)$, $z_0(\lambda)$ such that:
  - Each function is positive
  - Each function is a linear combination of $r_0(\lambda)$, $g_0(\lambda)$, and $b_0(\lambda)$.

CIE XYZ Definition

- CIE XYZ in terms of CIE RGB so that

$$\begin{bmatrix} x_0(\lambda) \\ y_0(\lambda) \\ z_0(\lambda) \end{bmatrix} = \mathbf{M} \begin{bmatrix} r_0(\lambda) \\ g_0(\lambda) \\ b_0(\lambda) \end{bmatrix}$$

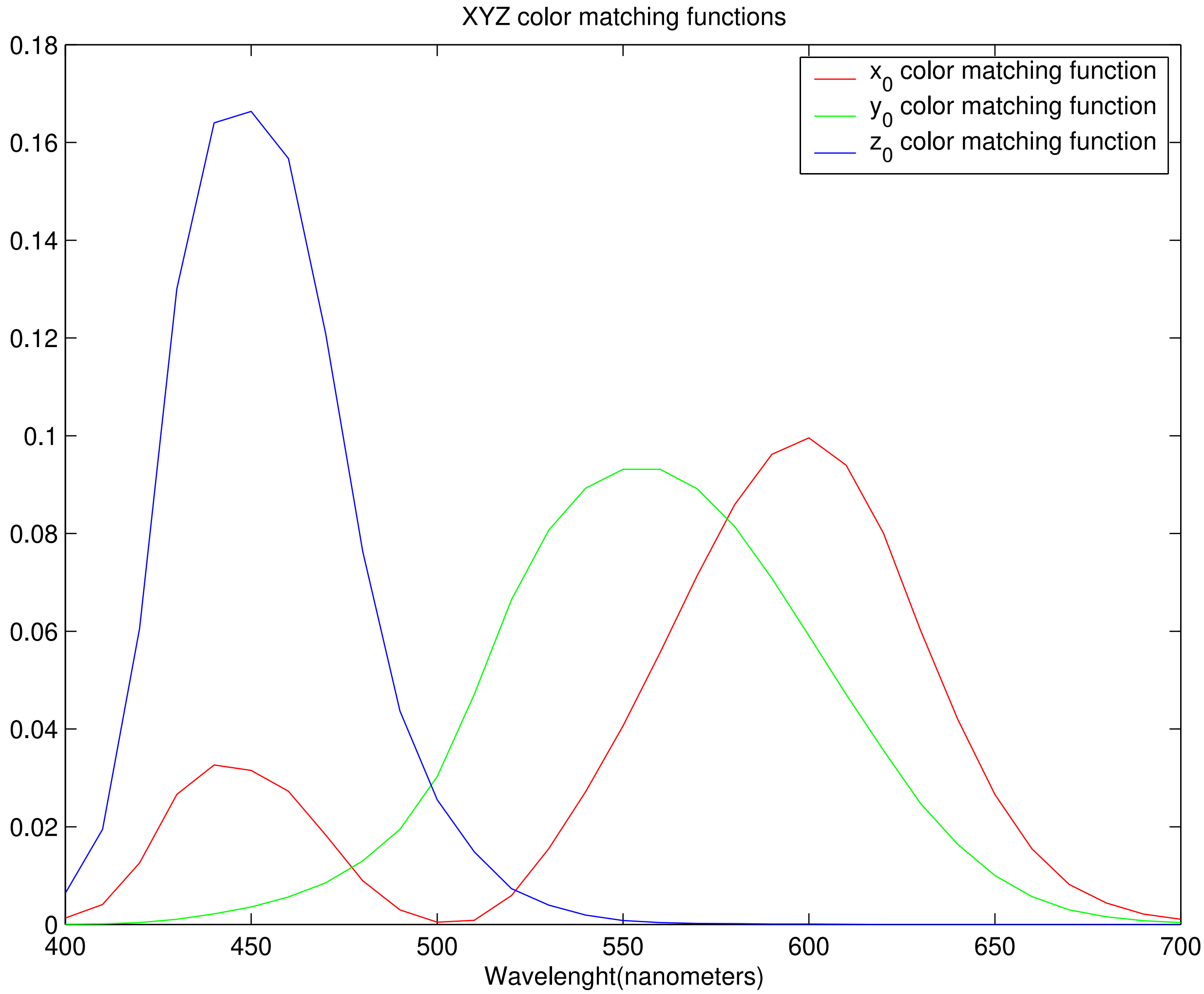
where

$$\mathbf{M} = \begin{bmatrix} 0.490 & 0.310 & 0.200 \\ 0.177 & 0.813 & 0.010 \\ 0.000 & 0.010 & 0.990 \end{bmatrix}$$

- This transformation is chosen so that

$$\begin{aligned} x_0(\lambda) &\geq 0 \\ y_0(\lambda) &\geq 0 \\ z_0(\lambda) &\geq 0 \end{aligned}$$

CIE XYZ Color Matching functions



XYZ Tristimulus Values

- The XYZ tristimulus values may be calculated as:

$$
\begin{aligned}
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} &= \int_0^\infty \begin{bmatrix} x_0(\lambda) \\ y_0(\lambda) \\ z_0(\lambda) \end{bmatrix} I(\lambda) d\lambda \\
&= \int_0^\infty \mathbf{M} \begin{bmatrix} r_0(\lambda) \\ g_0(\lambda) \\ b_0(\lambda) \end{bmatrix} I(\lambda) d\lambda \\
&= \mathbf{M} \int_0^\infty \begin{bmatrix} r_0(\lambda) \\ g_0(\lambda) \\ b_0(\lambda) \end{bmatrix} I(\lambda) d\lambda \\
&= \mathbf{M} \begin{bmatrix} r \\ g \\ b \end{bmatrix}
\end{aligned}
$$

XYZ/RGB Color Transformations

- So we have that XYZ can be computed from RGB as:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \mathbf{M} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

- Alternatively, RGB can be computed from XYZ as:

$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = \mathbf{M}^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- Comments:
  - Always use upper case letters for XYZ!
  - Y value represents luminance component of image
  - X is related to red.
  - Z is related to blue.

XYZ Color Primaries

- The XYZ color primaries are computed as

$$\begin{aligned} \text{Color} &= [\mathbf{X}, \mathbf{Y}, \mathbf{Z}] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \\ &= [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} r \\ g \\ b \end{bmatrix} \\ &= [\mathbf{R}, \mathbf{G}, \mathbf{B}]\, \mathbf{M}^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \end{aligned}$$

- So, theoretically

$$[\mathbf{X}, \mathbf{Y}, \mathbf{Z}] = [\mathbf{R}, \mathbf{G}, \mathbf{B}]\, \mathbf{M}^{-1}$$

where

$$\mathbf{M}^{-1} = \begin{bmatrix} 2.3644 & -0.8958 & -0.4686 \\ -0.5148 & 1.4252 & 0.0896 \\ 0.0052 & -0.0144 & 1.0092 \end{bmatrix}$$

Problem with XYZ Primaries

$$[\mathbf{X}, \mathbf{Y}, \mathbf{Z}] = [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} 2.3644 & -0.8958 & -0.4686 \\ -0.5148 & 1.4252 & 0.0896 \\ 0.0052 & -0.0144 & 1.0092 \end{bmatrix}$$

- Negative values in matrix imply that spectral distribution of XYZ primaries will be negative.
- The XYZ primaries can not be realized from physical combinations of CIE RGB.
- Fact: XYZ primaries are imaginary!

Alternative Choices for R,G,B Primaries

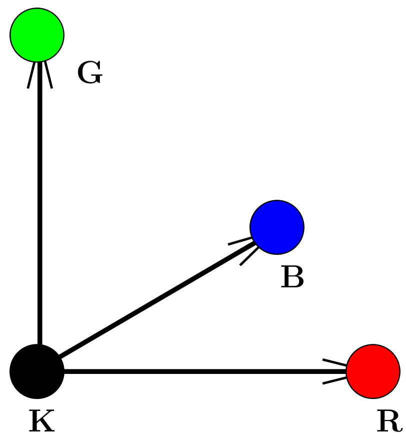
- Select your favorite $\mathbf{R}$, $\mathbf{G}$, and $\mathbf{B}$ color primaries.
  - These need not be CIE $\mathbf{R}$, $\mathbf{G}$, $\mathbf{B}$, but they should "look like" red, green, and blue.
  - For set of primaries $\mathbf{R}, \mathbf{G}, \mathbf{B}$, there must be a matrix transformation M such that

$$\begin{bmatrix} \mathbf{R} \\ \mathbf{G} \\ \mathbf{B} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \\ \mathbf{Z} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{R} \\ \mathbf{G} \\ \mathbf{B} \end{bmatrix} = \begin{bmatrix} X_r & Y_r & Z_r \\ X_g & Y_g & Z_g \\ X_b & Y_b & Z_b \end{bmatrix} \begin{bmatrix} \mathbf{X} \\ \mathbf{Y} \\ \mathbf{Z} \end{bmatrix}$$

- We will discuss alternative choices for $\mathbf{R}, \mathbf{G}, \mathbf{B}$ later
- The selection of $\mathbf{R}, \mathbf{G}, \mathbf{B}$ can impact:
  - The cost of rendering device/system
  - The "color gamut" of the device/system
  - System interoperability

Red, Green, Blue $(\mathrm{R}, \mathrm{G}, \mathrm{B})$ Color Vectors



- We can specify colors by a combination of

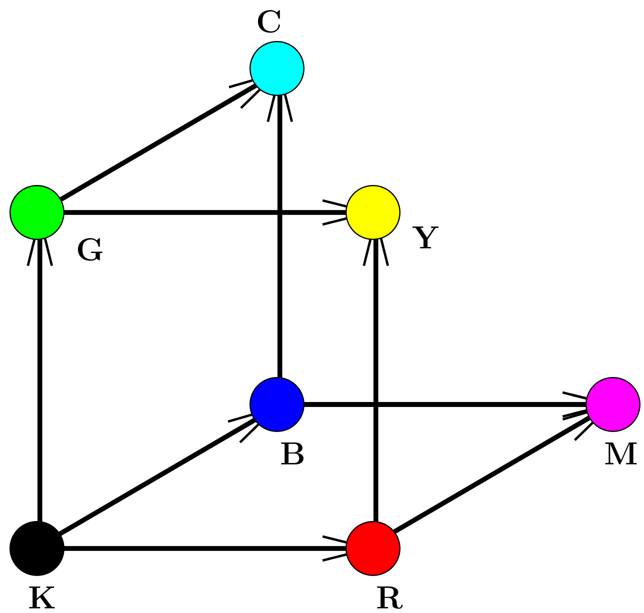
$$\begin{aligned} \text{Color} &= r\mathbf{R} + g\mathbf{G} + b\mathbf{B} \\ &= [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} r \\ g \\ b \end{bmatrix} \end{aligned}$$

  – $\mathbf{R}, \mathbf{G}, \mathbf{B}$ color primaries are basis vectors
  – (r, g, b) tristimulus values specify 3-D coordinates

- Each color can be specified by its (r, g, b) coordinates

$$\begin{aligned} \text{Red} = \mathbf{R} &\Leftrightarrow (r, g, b) = (1, 0, 0) \\ \text{Green} = \mathbf{G} &\Leftrightarrow (r, g, b) = (0, 1, 0) \\ \text{Blue} = \mathbf{B} &\Leftrightarrow (r, g, b) = (0, 0, 1) \end{aligned}$$

Cyan, Magenta, Yellow $(\mathrm{C}, \mathrm{M}, \mathrm{Y})$ Color Vectors

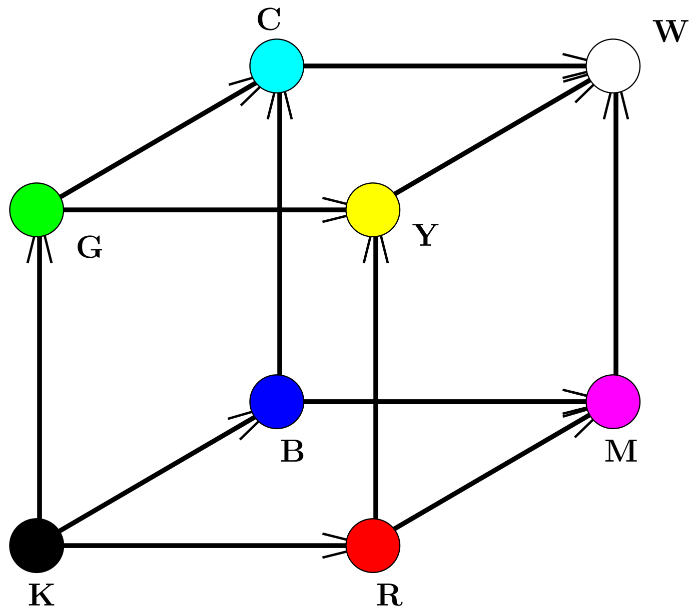


$$\text{Color} \; = \; [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

- Cyan, Magenta, and Yellow can each be specified by their (r, g, b) coordinates

$$\begin{aligned} \text{Cyan} = \mathbf{G} + \mathbf{B} \; &\Leftrightarrow \; (r, g, b) = (0, 1, 1) \\ \text{Magenta} = \mathbf{R} + \mathbf{B} \; &\Leftrightarrow \; (r, g, b) = (1, 0, 1) \\ \text{Yellow} = \mathbf{R} + \mathbf{G} \; &\Leftrightarrow \; (r, g, b) = (1, 1, 0) \end{aligned}$$

Full Color Cube



$$\text{White} \;=\; [\mathbf{R},\mathbf{G},\mathbf{B}]\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{aligned}
\text{White} = \mathbf{W} &\;\Leftrightarrow\; (r,g,b) = (1,1,1)\\
\text{Black} = \mathbf{K} &\;\Leftrightarrow\; (r,g,b) = (0,0,0)\\
\text{Red} = \mathbf{R} &\;\Leftrightarrow\; (r,g,b) = (1,0,0)\\
\text{Green} = \mathbf{G} &\;\Leftrightarrow\; (r,g,b) = (0,1,0)\\
\text{Blue} = \mathbf{B} &\;\Leftrightarrow\; (r,g,b) = (0,0,1)\\
\text{Cyan} = \mathbf{C} &\;\Leftrightarrow\; (r,g,b) = (0,1,1)\\
\text{Magenta} = \mathbf{M} &\;\Leftrightarrow\; (r,g,b) = (1,0,1)\\
\text{Yellow} = \mathbf{Y} &\;\Leftrightarrow\; (r,g,b) = (1,1,0)
\end{aligned}$$

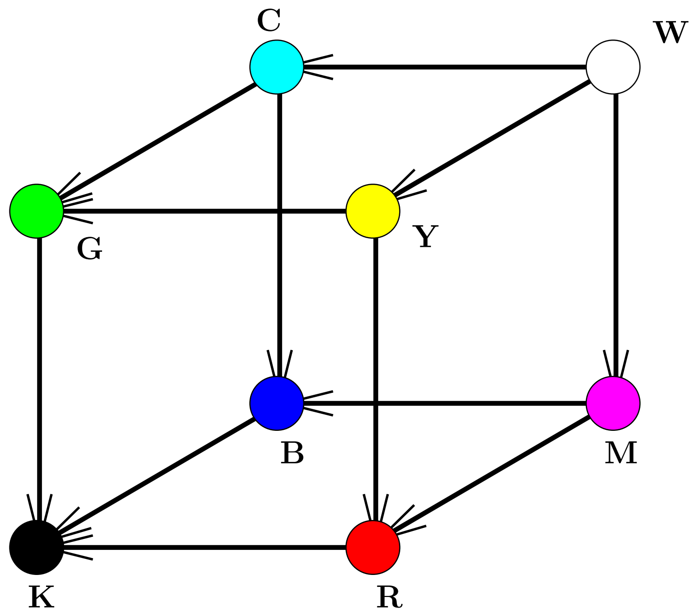
Subtractive Color Coordinates

$$
\begin{aligned}
& [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} r \\ g \\ b \end{bmatrix} \\
= \; & \mathbf{W} + [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} r \\ g \\ b \end{bmatrix} - \mathbf{W} \\
= \; & \mathbf{W} + [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} r \\ g \\ b \end{bmatrix} - [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \\
= \; & \mathbf{W} - [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} 1-r \\ 1-g \\ 1-b \end{bmatrix} \\
= \; & \mathbf{W} - [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} c \\ m \\ y \end{bmatrix}
\end{aligned}
$$

where

$$
\begin{bmatrix} c \\ m \\ y \end{bmatrix} \triangleq \begin{bmatrix} 1-r \\ 1-g \\ 1-b \end{bmatrix}
$$

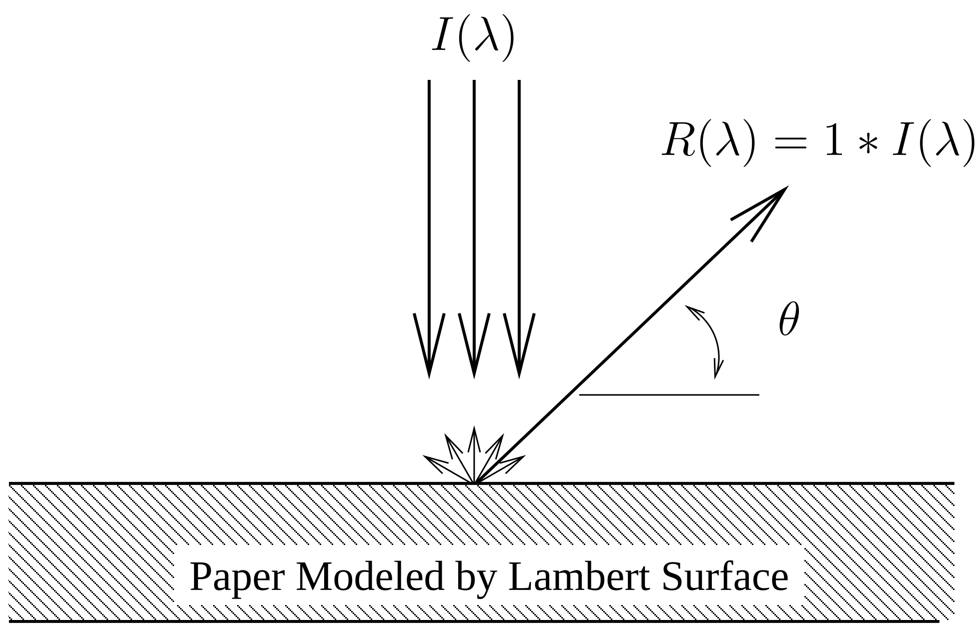
C, M, Y Color Coordinates



$$\text{Color} \;=\; \mathbf{W} - [\mathbf{R}, \mathbf{G}, \mathbf{B}] \begin{bmatrix} c \\ m \\ y \end{bmatrix}$$

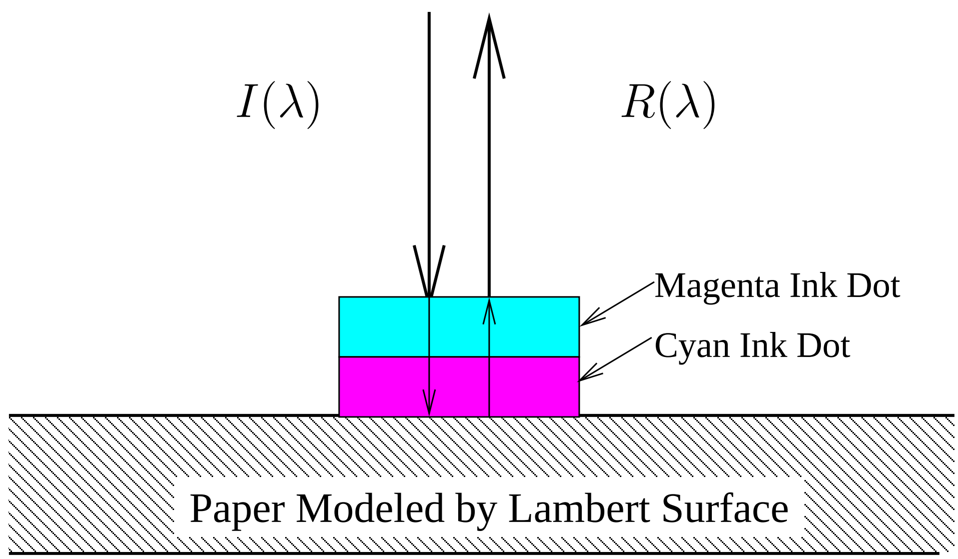
- This is called a subtractive color system because (c, m, y) coordinates subtract color from white
- Subtractive color is important in:
  - Printing
  - Paints and dyes
  - Films and transparencies

Light Reflection from Lambert Surface



- White Lambert Surface
- Reflected luminance is independent of:
  - Viewing angle (θ)
  - Wavelength (λ)

Effect of Ink on Reflected Light



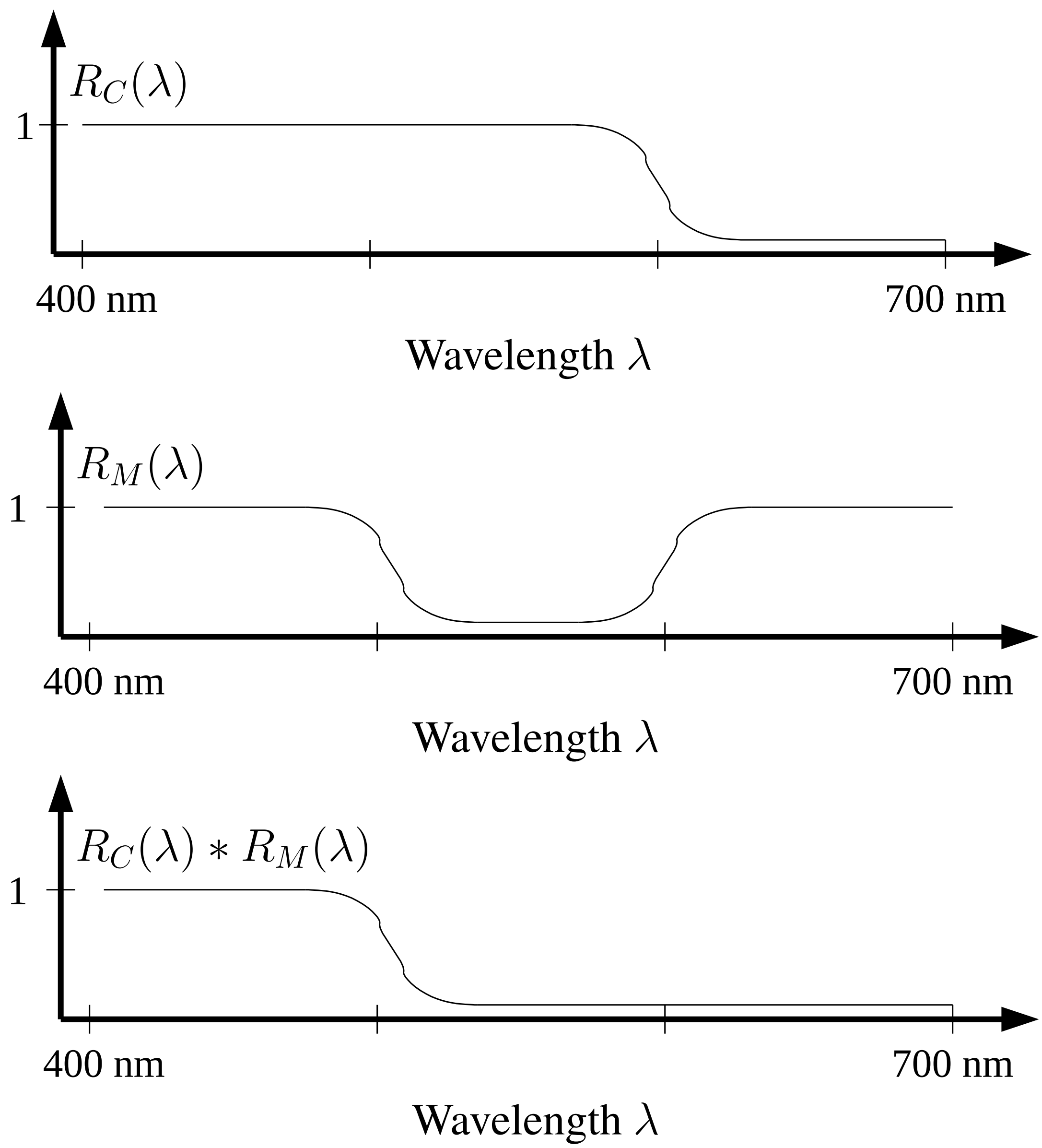
- Reflected light is given by

$$R(\lambda) = R_C(\lambda) R_M(\lambda) I(\lambda)$$

  - Reflected light is from by product of functions
  - Inks interact nonlinearly (multiplication versus addition)

- What color is formed by magenta and cyan ink?

Simplified Spectral Reflectance of Ink



- Reflected light appears blue
  - Both green and red components have been removed
  - Each ink subtracts colors from the illuminant