What is Color?

- Color is a human perception (a percept).
- Color is not a physical property...
- But, it is related to the light spectrum of a stimulus.
Can We Measure the Percept of Color?

- Semantic names - red, green, blue, orange, yellow, etc.
- These color semantics are largely culturally invariant, but not precisely.
- Currently, there is no accurate model for predicting perceived color from the light spectrum of a stimulus.
- Currently, no one has an accurate model for predicting the percept of color.
Can We Tell if Two Colors are the Same?

- Two colors are the same if they match at all spectral wavelengths.
- However, we will see that two colors are also the same if they match on a 3 dimensional subspace.
- The values on this three dimensional subspace are called tristimulus values.
- Two colors that match are called metamers.
Matching a Color Patch

• Experimental set up:
  
  – Form a reference color patch with a known spectral distribution.

    Reference Color ⇒ \( I(\lambda) \)

  – Form a second adjustable color patch by adding light with three different spectral distributions.

    Red ⇒ \( I_r(\lambda) = R \)

    Green ⇒ \( I_g(\lambda) = G \)

    Blue ⇒ \( I_b(\lambda) = B \)

  – Control the amplitude of each component with three individual positive constants \( r^+, g^+, \) and \( b^+ \).

  – The total spectral content of the adjustable patch is then

    \[
    r^+ I_r(\lambda) + g^+ I_g(\lambda) + b^+ I_b(\lambda) .
    \]

• Choose \((r^+, g^+, b^+)\) to match the two color patches.
Simple Color Matching with Primaries

- Choose \((r^+, g^+, b^+\)) to match the two color patches.
- The values of \((r, g, b)\) must be positive!
- Definitions:
  - \(R, G,\) and \(B\) are known as color primaries.
  - \(r^+, g^+,\) and \(b^+\) are known as tristimulus values.
- Problem:
  - Some colors can not be matched, because they are too “saturated”.
  - These colors result in values of \(r^+, g^+,\) or \(b^+\) which are 0.
  - How can we generate negative values for \(r^+, g^+,\) or \(b^+\)?
Improved Color Matching with Primaries

- Add color primaries to reference color!
- This is equivalent to subtracting them from adjustable patch.
- Equivalent tristimulus values are:
  \[ r = r^+ - r^- \]
  \[ g = g^+ - g^- \]
  \[ b = b^+ - b^- \]
- In this case, \( r, g, \) and \( b \) can be both positive and negative.
- All colors may be matched.
Grassman’s Law

• Grassman’s law: Color perception is a 3 dimensional linear space.

• Superposition:
  – Let $I_1(\lambda)$ have tristimulus values $(r_1, g_1, b_1)$, and let $I_2(\lambda)$ have tristimulus values $(r_2, g_2, b_2)$.
  – Then $I_3(\lambda) = I_1(\lambda) + I_2(\lambda)$ has tristimulus values of
    $$(r_3, g_3, b_3) = (r_1, g_1, b_1) + (r_2, g_2, b_2)$$

• This implies that tristimulus values can be computed with a linear functional of the form
  $$r = \int_0^\infty r_0(\lambda) I(\lambda) d\lambda$$
  $$g = \int_0^\infty g_0(\lambda) I(\lambda) d\lambda$$
  $$b = \int_0^\infty b_0(\lambda) I(\lambda) d\lambda$$

  for some functions $r_0(\lambda)$, $g_0(\lambda)$, and $b_0(\lambda)$.

• Definition: $r_0(\lambda)$, $g_0(\lambda)$, and $b_0(\lambda)$ are known as color matching functions.
Measuring Color Matching Functions

• A pure color at wavelength $\lambda_0$ is known as a line spectrum. It has spectral distribution

$$I(\lambda) = \delta(\lambda - \lambda_0).$$

Pure colors can be generated using a laser or a very narrow band spectral filter.

• When the reference color is such a pure color, then the tristimulus values are given by

$$r = \int_0^\infty r_0(\lambda) \delta(\lambda - \lambda_0) d\lambda = r_0(\lambda_0)$$

$$g = \int_0^\infty g_0(\lambda) \delta(\lambda - \lambda_0) d\lambda = g_0(\lambda_0)$$

$$b = \int_0^\infty b_0(\lambda) \delta(\lambda - \lambda_0) d\lambda = b_0(\lambda_0)$$

• Method for Measuring Color Matching Functions:

  – Color match to a reference color generated by a pure spectral source at wavelength $\lambda_0$.
  – Record the tristimulus values of $r_0(\lambda_0)$, $g_0(\lambda_0)$, and $b_0(\lambda_0)$ that you obtain.
  – Repeat for all values of $\lambda_0$. 
CIE Standard RGB Color Matching Functions

- An organization call Commission Internationale de l’Eclairage (CIE) defined all practical standards for color measurements (colorimetry).

- CIE 1931 Standard $2^\circ$ Observer:
  - Uses color patches that subtended $2^\circ$ of visual angle.
  - $R, G, B$ color primaries are defined by pure line spectra (delta functions in wavelength) at 700nm, 546.1nm, and 435.8nm.
  - Reference color is a spectral line at wavelength $\lambda$.

- CIE 1965 $10^\circ$ Observer: A slightly different standard based on a $10^\circ$ reference color patch and a different measurement technique.
RGB Color Matching Functions for CIE Standard 2° Observer

![Diagram of RGB color matching functions]

- The color matching functions are then given by

\[
\begin{align*}
    r_0(\lambda) &= r^+ - r^- \\
    g_0(\lambda) &= g^+ - g^- \\
    b_0(\lambda) &= b^+ - b^- 
\end{align*}
\]

where \( \lambda \) is the wavelength of the reference line spectrum.
RGB Color Matching Functions for CIE Standard $2^\circ$ Observer

- Plotting the values of $r_0(\lambda)$, $g_0(\lambda)$, and $b_0(\lambda)$ results in the following.

- Notice that the functions take on negative values.
Review of Colorimetry Concepts

1. R, G, B are color primaries used to generate colors.

2. \((r, g, b)\) are tristimulus values used as weightings for the primaries.

\[
\text{Color} = rR + gG + bB
\]

\[
= \begin{bmatrix} r \\ g \\ b \end{bmatrix}
= [R, G, B] \begin{bmatrix} r \\ g \\ b \end{bmatrix}
\]

3. \((r_0(\lambda), g_0(\lambda), b_0(\lambda))\) are the color matching functions used to compute the tristimulus values.

\[
r = \int_0^\infty r_0(\lambda) I(\lambda)d\lambda
\]

\[
g = \int_0^\infty g_0(\lambda) I(\lambda)d\lambda
\]

\[
b = \int_0^\infty b_0(\lambda) I(\lambda)d\lambda
\]

- How are the color matching functions scaled?
Scaling of Color Matching Functions

- Color matching functions are scaled to have unit area
  \[ \int_0^\infty r_0(\lambda) d\lambda = 1 \]
  \[ \int_0^\infty g_0(\lambda) d\lambda = 1 \]
  \[ \int_0^\infty b_0(\lambda) d\lambda = 1 \]

- Color “white”
  - Has approximately equal energy at all wavelengths
  - \( I(\lambda) = 1 \)
  - White \( \iff (r, g, b) = (1, 1, 1) \)
  - Known as equal energy (EE) white
  - We will talk about this more later
Problems with CIE RGB

• Some colors generate negative values of $(r, g, b)$.

• This results from the fact that the color matching functions $r_0(\lambda), g_0(\lambda), b_0(\lambda)$ can be negative.

• The color primaries corresponding to CIE RGB are very difficult to reproduce. (pure spectral lines)

• Partial solution: Define new color matching functions $x_0(\lambda), y_0(\lambda), z_0(\lambda)$ such that:
  
  – Each function is positive

  – Each function is a linear combination of $r_0(\lambda), g_0(\lambda), b_0(\lambda)$.  

CIE XYZ Definition

- CIE XYZ in terms of CIE RGB so that

\[
\begin{bmatrix}
    x_0(\lambda) \\
    y_0(\lambda) \\
    z_0(\lambda)
\end{bmatrix} = \mathbf{M}
\begin{bmatrix}
    r_0(\lambda) \\
    g_0(\lambda) \\
    b_0(\lambda)
\end{bmatrix}
\]

where

\[
\mathbf{M} = \begin{bmatrix}
    0.490 & 0.310 & 0.200 \\
    0.177 & 0.813 & 0.010 \\
    0.000 & 0.010 & 0.990
\end{bmatrix}
\]

- This transformation is chosen so that

\[
x_0(\lambda) \geq 0
\]

\[
y_0(\lambda) \geq 0
\]

\[
z_0(\lambda) \geq 0
\]
CIE XYZ Color Matching functions

![XYZ color matching functions](image)
XYZ Tristimulus Values

- The XYZ tristimulus values may be calculated as:

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = \int_{0}^{\infty} \begin{bmatrix}
x_0(\lambda) \\
y_0(\lambda) \\
z_0(\lambda)
\end{bmatrix} I(\lambda) d\lambda
\]

\[
= \int_{0}^{\infty} M \begin{bmatrix}
r_0(\lambda) \\
g_0(\lambda) \\
b_0(\lambda)
\end{bmatrix} I(\lambda) d\lambda
\]

\[
= M \int_{0}^{\infty} \begin{bmatrix}
r_0(\lambda) \\
g_0(\lambda) \\
b_0(\lambda)
\end{bmatrix} I(\lambda) d\lambda
\]

\[
= M \begin{bmatrix}
r \\
g \\
b
\end{bmatrix}
\]
XYZ/RGB Color Transformations

• So we have that XYZ can be computed from RGB as:

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
= \mathbf{M}
\begin{bmatrix}
r \\
g \\
b
\end{bmatrix}
\]

• Alternatively, RGB can be computed from XYZ as:

\[
\begin{bmatrix}
r \\
g \\
b
\end{bmatrix}
= \mathbf{M}^{-1}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\]

• Comments:

  – Always use upper case letters for XYZ!
  – \( Y \) value represents luminance component of image
  – \( X \) is related to red.
  – \( Z \) is related to blue.
XYZ Color Primaries

• The XYZ color primaries are computed as

\[
\text{Color} = [X, Y, Z] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
\]

\[
= [R, G, B] \begin{bmatrix} r \\ g \\ b \end{bmatrix}
\]

\[
= [R, G, B] M^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}
\]

• So, theoretically

\[
[X, Y, Z] = [R, G, B] M^{-1}
\]

where

\[
M^{-1} = \begin{bmatrix} 2.3644 & -0.8958 & -0.4686 \\ -0.5148 & 1.4252 & 0.0896 \\ 0.0052 & -0.0144 & 1.0092 \end{bmatrix}
\]
Problem with XYZ Primaries

\[
\begin{bmatrix}
X, Y, Z
\end{bmatrix} = \begin{bmatrix}
R, G, B
\end{bmatrix}
\begin{bmatrix}
2.3644 & -0.8958 & -0.4686 \\
-0.5148 & 1.4252 & 0.0896 \\
0.0052 & -0.0144 & 1.0092
\end{bmatrix}
\]

- Negative values in matrix imply that spectral distribution of XYZ primaries will be negative.
- The XYZ primaries can not be realized from physical combinations of CIE RGB.
- Fact: XYZ primaries are imaginary!
Alternative Choices for R,G,B Primaries

• Select your favorite R, G, and B color primaries.
  – These need not be CIE R, G, B, but they should “look like” red, green, and blue.
  – For set of primaries R, G, B, there must be a matrix transformation $M$ such that

$$
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = M
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
$$

$$
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = \begin{bmatrix}
X_r & Y_r & Z_r \\
X_g & Y_g & Z_g \\
X_b & Y_b & Z_b
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
$$

• We will discuss alternative choices for R, G, B later

• The selection of R, G, B can impact:
  – The cost of rendering device/system
  – The “color gamut” of the device/system
  – System interoperability
Red, Green, Blue (R, G, B) Color Vectors

- We can specify colors by a combination of

\[
\text{Color} = rR + gG + bB
\]

\[
= [R, G, B] \begin{bmatrix} r \\ g \\ b \end{bmatrix}
\]

- R, G, B color primaries are basis vectors
- \((r, g, b)\) tristimulus values specify 3-D coordinates

- Each color can be specified by its \((r, g, b)\) coordinates

\[
\text{Red} = R \iff (r, g, b) = (1, 0, 0)
\]
\[
\text{Green} = G \iff (r, g, b) = (0, 1, 0)
\]
\[
\text{Blue} = B \iff (r, g, b) = (0, 0, 1)
\]
Cyan, Magenta, Yellow (C, M, Y) Color Vectors

\[ \text{Color} = [R, G, B] \begin{bmatrix} r \\ g \\ b \end{bmatrix} \]

- Cyan, Magenta, and Yellow can each be specified by their \((r, g, b)\) coordinates

  - \(\text{Cyan} = G + B \iff (r, g, b) = (0, 1, 1)\)
  - \(\text{Magenta} = R + B \iff (r, g, b) = (1, 0, 1)\)
  - \(\text{Yellow} = R + G \iff (r, g, b) = (1, 1, 0)\)
### Full Color Cube

![Full Color Cube Diagram]

<table>
<thead>
<tr>
<th>Color</th>
<th>RGB Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>$[R, G, B] = [1, 1, 1]$</td>
</tr>
<tr>
<td>Black</td>
<td>$(r, g, b) = (0, 0, 0)$</td>
</tr>
<tr>
<td>Red</td>
<td>$(r, g, b) = (1, 0, 0)$</td>
</tr>
<tr>
<td>Green</td>
<td>$(r, g, b) = (0, 1, 0)$</td>
</tr>
<tr>
<td>Blue</td>
<td>$(r, g, b) = (0, 0, 1)$</td>
</tr>
<tr>
<td>Cyan</td>
<td>$(r, g, b) = (0, 1, 1)$</td>
</tr>
<tr>
<td>Magenta</td>
<td>$(r, g, b) = (1, 0, 1)$</td>
</tr>
<tr>
<td>Yellow</td>
<td>$(r, g, b) = (1, 1, 0)$</td>
</tr>
</tbody>
</table>
**Subtractive Color Coordinates**

\[
\begin{bmatrix}
R, G, B
\end{bmatrix}
\begin{bmatrix}
r \\
g \\
b
\end{bmatrix}
= W + \begin{bmatrix}
R, G, B
\end{bmatrix}
\begin{bmatrix}
r \\
g \\
b
\end{bmatrix} - W
= W + \begin{bmatrix}
R, G, B
\end{bmatrix}
\begin{bmatrix}
r \\
g \\
b
\end{bmatrix} - \begin{bmatrix}
R, G, B
\end{bmatrix}
\begin{bmatrix}
1 \\
1
\end{bmatrix}
= W - \begin{bmatrix}
R, G, B
\end{bmatrix}
\begin{bmatrix}
1 - r \\
1 - g \\
1 - b
\end{bmatrix}
= W - \begin{bmatrix}
R, G, B
\end{bmatrix}
\begin{bmatrix}
c \\
m \\
y
\end{bmatrix}
\]

where

\[
\begin{bmatrix}
c \\
m \\
y
\end{bmatrix} \triangleq \begin{bmatrix}
1 - r \\
1 - g \\
1 - b
\end{bmatrix}
\]
$C$, $M$, $Y$ Color Coordinates

Color \( = \) \( W - [R, G, B] \begin{bmatrix} c \\ m \\ y \end{bmatrix} \)

- This is called a subtractive color system because \((c, m, y)\) coordinates subtract color from white
- Subtractive color is important in:
  - Printing
  - Paints and dyes
  - Films and transparencies
Light Reflection from Lambert Surface

\[ R(\lambda) = 1 \times I(\lambda) \]

- White Lambert Surface
- Reflected luminance is independent of:
  - Viewing angle (\(\theta\))
  - Wavelength (\(\lambda\))
Effect of Ink on Reflected Light

Reflected light is given by

\[ R(\lambda) = R_C(\lambda)R_M(\lambda)I(\lambda) \]

- Reflected light is from by product of functions
- Inks interact nonlinearly (multiplication versus addition)

- What color is formed by magenta and cyan ink?
- Reflected light appears blue
  - Both green and red components have been removed
  - Each ink subtracts colors from the illuminant