Region Segmentation

- Connected components analysis often results in many small disjointed regions.
- A connection (or break) at a single pixel can split (or merge) entire regions.
- There are three basic approaches to segmentation:
  - Region Merging - recursively merge regions that are similar.
    
    ![Region Merging Diagram](image)
  
  - Region Splitting - recursively divide regions that are heterogeneous.
    
    ![Region Splitting Diagram](image)
  
  - Split and merge - iteratively split and merge regions to form the “best” segmentation.
Hierarchical Clustering

• Clustering refers to techniques for separating data samples into sets with distinct characteristics.

• Clustering methods are analogous to segmentation methods.
  – Agglomerative clustering - “bottom up” procedure for recursively merging clusters ⇒ analogous to region merging
  – Divisive clustering - “top down” procedure for recursively splitting clusters ⇒ analogous to region splitting
Image Regions and Partitions

• Let \( R_m \subset S \) denote a region of the image where \( m \in \mathcal{M} \).
• We say that \( \{ R_m | m \in \mathcal{M} \} \) partitions the image if
  
  For all \( m \neq k \), \( R_m \cap R_k = \emptyset \)
  
  \( \bigcup_{m \in \mathcal{M}} R_m = S \)

• Each region \( R_m \) has features that characterize it.
Typical Region Features

• Color
  – Mean RGB value
  – 1-D color histograms in R, G, and B
  – 3-D color histogram in (R,G,B)

• Texture
  – Spatial autocorrelation
  – Joint probability distribution for neighboring pixels (e.g. the spatial co-occurrence matrix)
  – Wavelet transform coefficients

• Shape
  – Number of pixels
  – Width and height attributes
  – Boundary smoothness attributes
  – Adjacent region labels
Recursive Feature Computation

• Any two regions may be merged into a new region.

\[ R_{new} = R_k \cup R_l \]

• Let \( f_n = f(R_n) \in \mathbb{R}^k \) be a \( k \) dimensional feature vector extracted from the region \( R_n \).

• Ideally, the features of merged regions may be computed without reference to the original pixels in the region.

\[ f(R_k \cup R_l) = f(R_k) \oplus f(R_l) \]
\[ f_{new} = f_k \oplus f_l \]

here \( \oplus \) denotes some operation on the values of the two feature vectors.
Example of Recursive Feature Computation

Example: Let \( f(R_k) = (N_k, \mu_k, c_k) \) where

\[
N_k = |R_k| \\
\mu_k = \frac{1}{N_k} \sum_{s \in R_k} x_s \\
c_k = \frac{1}{N_k} \sum_{s \in R_k} s
\]

We may compute the region features for \( R_{\text{new}} = R_k \cup R_l \) using the recursions

\[
N_{\text{new}} = N_k + N_l \\
\mu_{\text{new}} = \frac{N_k \mu_k + N_l \mu_l}{N_{\text{new}}} \\
c_{\text{new}} = \frac{N_k c_k + N_l c_l}{N_{\text{new}}}
\]
Recursive Merging

• Define a distance function between regions. In general, this function has the form

\[ d_{k,l} = D(R_k, R_l) \geq 0 \]

• Ideally, \( D(R_k, R_l) \) is **only** a function of the feature vectors \( f_k \) and \( f_l \).

\[ d_{k,l} = D(f_k, f_l) \geq 0 \]

• Then merge regions with minimum distance.
Example of Merging Criteria

- Distance between color means
  \[ d_{k,l} = \frac{N_k}{N_{\text{new}}} |\mu_k - \mu_{\text{new}}|^2 + \frac{N_l}{N_{\text{new}}} |\mu_l - \mu_{\text{new}}|^2 \]

- Distance between region centers
  \[ d_{k,l} = \frac{N_k}{N_{\text{new}}} |c_k - c_{\text{new}}|^2 + \frac{N_l}{N_{\text{new}}} |c_l - c_{\text{new}}|^2 \]

- Distance formed by a weighted combination of the two
  \[ d_{k,l} = \alpha \left( \frac{N_k}{N_{\text{new}}} |\mu_k - \mu_{\text{new}}|^2 + \frac{N_l}{N_{\text{new}}} |\mu_l - \mu_{\text{new}}|^2 \right) + \beta \left( \frac{N_k}{N_{\text{new}}} |c_k - c_{\text{new}}|^2 + \frac{N_l}{N_{\text{new}}} |c_l - c_{\text{new}}|^2 \right) \]
Recursive Merging Algorithm

- Define a distance function between regions
  \[ d_{k,l} = D(f(R_k), f(R_l)) > 0 \]

Repeat until \(|\mathcal{M}| = 1\) {
  - Determine the minimum distance regions
    \[ (k^*, l^*) = \arg \min_{k, l \in \mathcal{M}} \{d_{k,l}\} \]
  - Merge the minimum distance regions
    \[ R_{k^*} \leftarrow R_{k^*} \cup R_{l^*} \]
  - Remove unused region
    \[ \mathcal{M} \leftarrow \mathcal{M} - \{l^*\} \]
}

- This recursion generates a binary tree.
Merging Hierarchy and Order Identification

- Clustering can be terminated when the distance exceeds a threshold

\[ d_{k*,l*} > \text{Threshold} \Rightarrow \text{Stop clustering} \]

- Different thresholds result in different numbers of clusters.