

Magnetic Resonance Imaging (MRI)

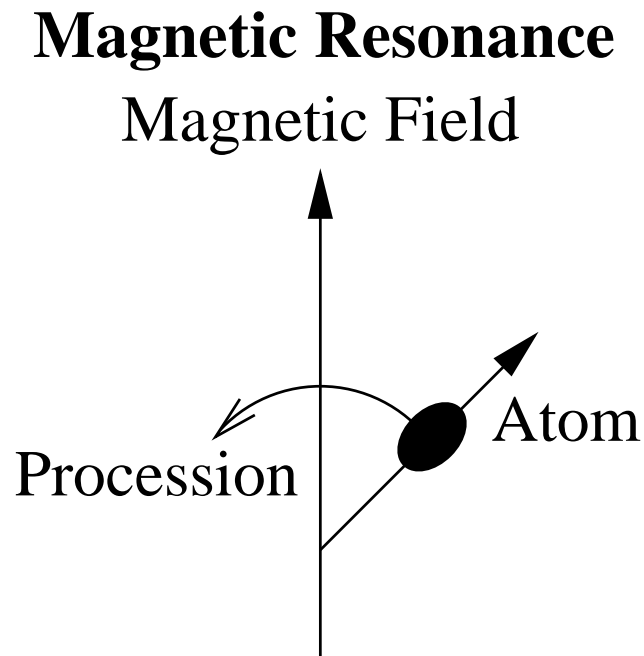


- Can be very high resolution
- No radiation exposure
- Very flexible and programmable
- Tends to be expensive, noisy, slow



MRI Attributes

- Based on magnetic resonance effect in atomic species
- Does not require any ionizing radiation
- Numerous modalities
 - Conventional anatomical scans
 - Functional MRI (fMRI)
 - MRI Tagging
- Image formation
 - RF excitation of magnetic resonance modes
 - Magnetic field gradients modulate resonance frequency
 - Reconstruction computed with inverse Fourier transform
 - Fully programmable
 - Requires an enormous (and very expensive) superconducting magnet



- Atom will precess at the Larmor frequency:

$$\omega_o = LM$$

- Quantities of importance:

M - magnitude of ambient magnetic field

ω_o - frequency of precession (radians per second)

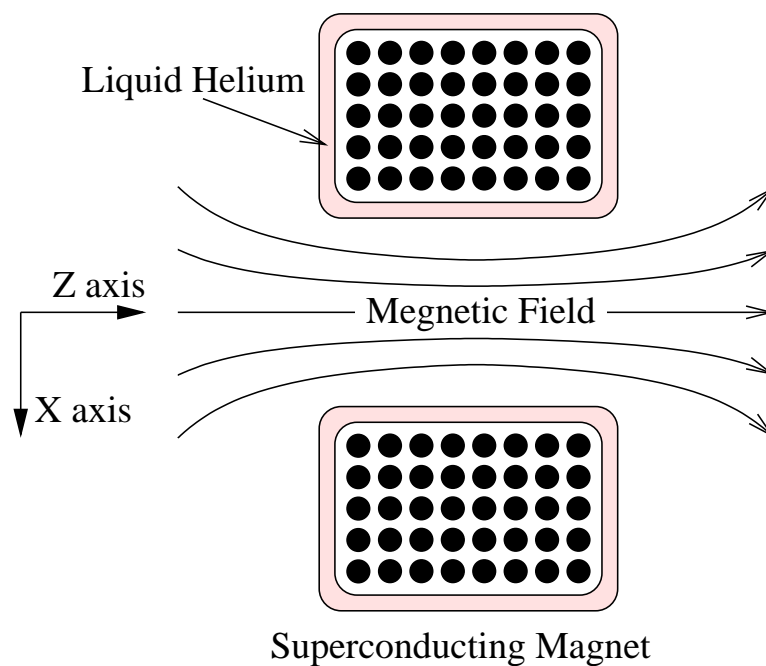
L - Larmor constant. Depends on choice of atom

- Typical values:

For hydrogen: 42.58 MHz/T; 1.5 T \Rightarrow 63.87 MHz; 3 T \Rightarrow 127.74 MHz

For carbon 13: 10.705 MHz/T; 1.5 T \Rightarrow 16.06 MHz; 3 T \Rightarrow 32.12 MHz

The MRI Magnet



- Large super-conducting magnet
 - Uniform field within bore
 - Very large static magnetic field of M_o

Magnetic Field Gradients

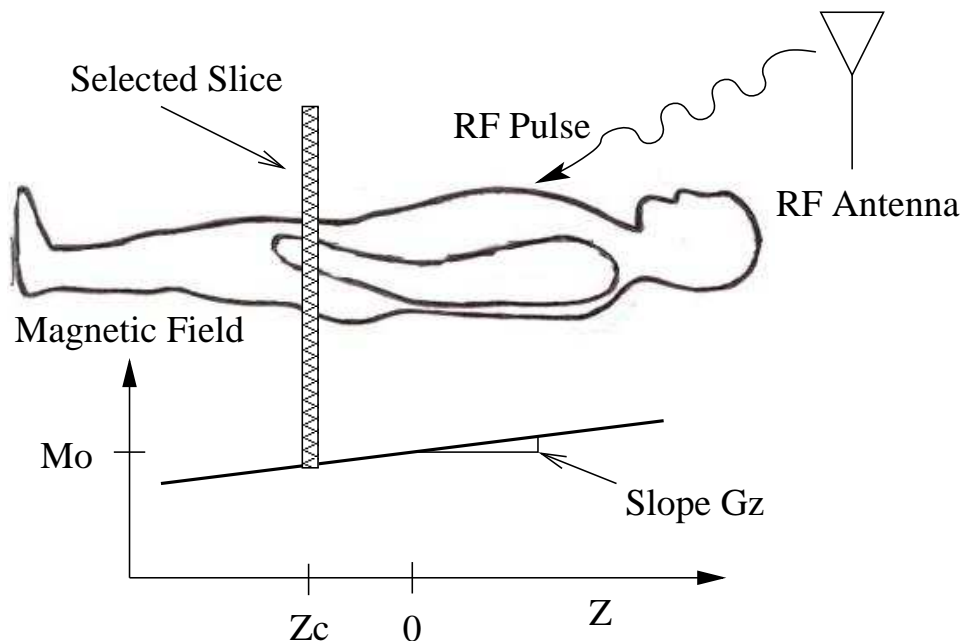
- Magnetic field **magnitude** at the location (x, y, z) has the form

$$M(x, y, z) = M_o + xG_x + yG_y + zG_z$$

- G_x , G_y , and G_z control magnetic field gradients
 - Gradients can be changed with time
 - Gradients are small compared to M_o
- For time varying gradients

$$M(x, y, z, t) = M_o + xG_x(t) + yG_y(t) + zG_z(t)$$

MRI Slice Select



- Design RF pulse to excite protons in single slice
 - Turn off x and y gradients, i.e. $G_x = G_y = 0$.
 - Set z gradient to fix positive value, $G_z > 0$.
 - Use the fact that resonance frequency is given by

$$\omega = L (M_o + zG_z) .$$

Slice Select Pulse Design

- Design parameters
 - Slice center = z_c .
 - Slice thickness = Δz .
- Slice centered at $z_c \Rightarrow$ pulse center frequency

$$f_c = \frac{LM_o}{2\pi} + \frac{z_c LG_z}{2\pi} = f_o + \frac{z_c LG_z}{2\pi} .$$

- Slice thickness $\Delta z \Rightarrow$ pulse bandwidth

$$\Delta f = \frac{\Delta z LG_z}{2\pi} .$$

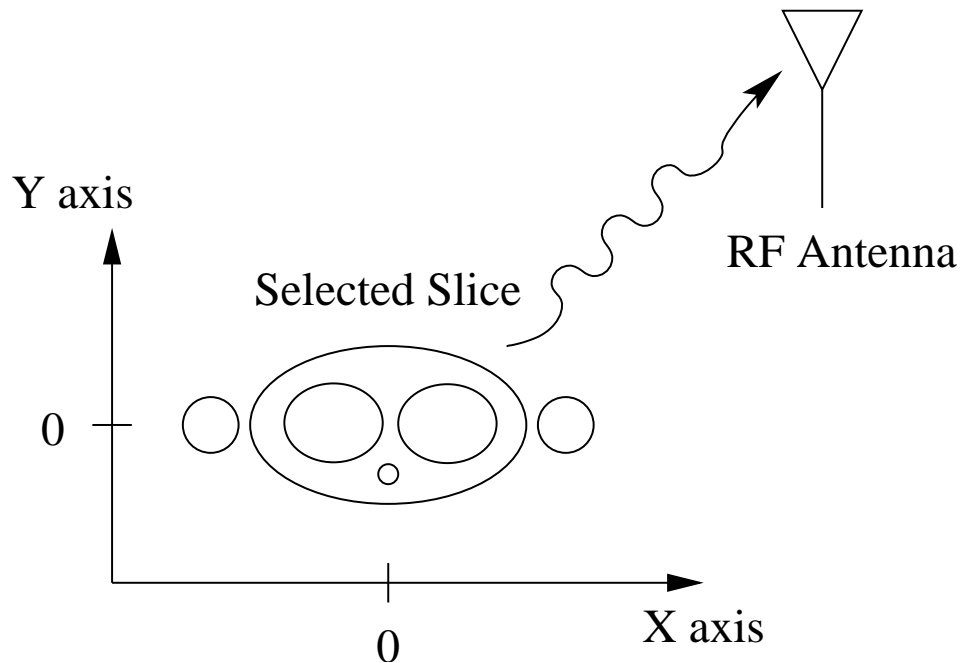
- Using these parameters, the pulse is given by

$$s(t) = e^{j2\pi f_c t} \text{sinc}(t\Delta f)$$

and its CTFT is given by

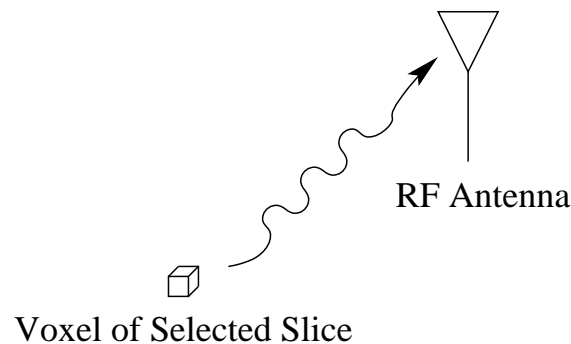
$$S(f) = \text{rect}\left(\frac{(f - f_c)}{\Delta f}\right)$$

How Do We Imaging Selected Slice?



- Precessing atoms radiate electromagnetic energy at RF frequencies
- Strategy
 - Vary magnetic gradients along x and y axes
 - Measure received RF signal
 - Reconstruct image from RF measurements

Signal from a Single Voxel



- RF signal from a single voxel has the form

$$r(x, y, t) = f(x, y)e^{j\phi(t)}$$

$f(x, y)$ voxel dependent weighting

- Depends on properties of material in voxel
- Quantity of interest
- Typically “weighted” by T1, T2, or T2*

$\phi(t)$ phase of received signal

- Can be modulated using G_x and G_y magnetic field gradients
- We assume that $\phi(0) = 0$

Analysis of Phase

- Frequency = time derivative of phase

$$\frac{d\phi(t)}{dt} = L M(x, y, t)$$

$$\phi(t) = \int_0^t L M(x, y, \tau) d\tau$$

$$= \int_0^t L M_o + x L G_x(\tau) + y L G_y(\tau) d\tau$$

$$= \omega_o t + x k_x(t) + y k_y(t)$$

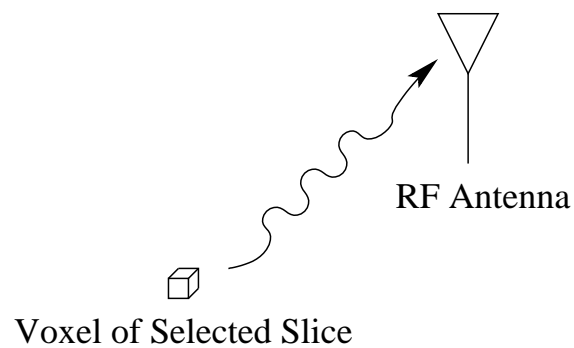
where we define

$$\omega_o = L M_o$$

$$k_x(t) = \int_0^t L G_x(\tau) d\tau$$

$$k_y(t) = \int_0^t L G_y(\tau) d\tau$$

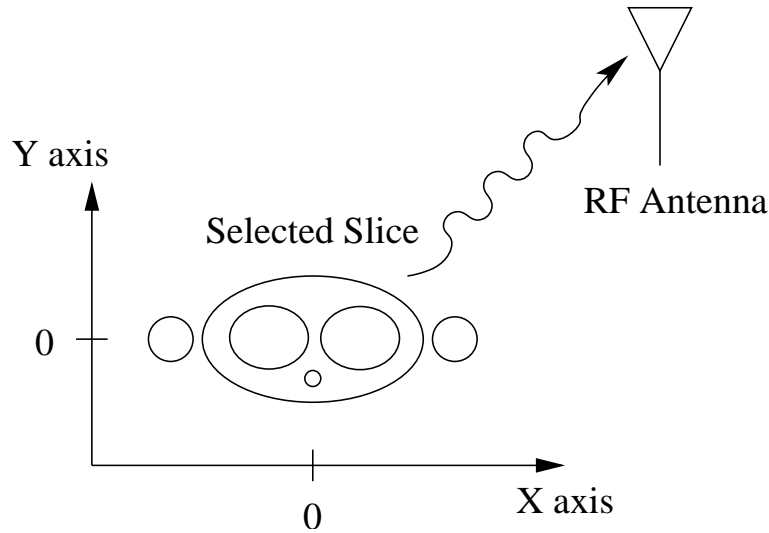
Received Signal from Voxel



- RF signal from a single voxel has the form

$$\begin{aligned} r(t) &= f(x, y)e^{j\phi(t)} \\ &= f(x, y)e^{j(\omega_0 t + xk_x(t) + yk_y(t))} \\ &= f(x, y)e^{j\omega_0 t} e^{j(xk_x(t) + yk_y(t))} \end{aligned}$$

Received Signal from Selected Slice



- RF signal from the complete slice is given by

$$\begin{aligned}
 R(t) &= \int_{\mathbb{R}} \int_{\mathbb{R}} r(x, y, t) dx dy \\
 &= \int_{\mathbb{R}} \int_{\mathbb{R}} f(x, y) e^{j\omega_0 t} e^{j(xk_x(t) + yk_y(t))} dx dy \\
 &= e^{j\omega_0 t} \int_{\mathbb{R}} \int_{\mathbb{R}} f(x, y) e^{j(xk_x(t) + yk_y(t))} dx dy \\
 &= e^{j\omega_0 t} F\left(-\frac{k_x(t)}{2\pi}, -\frac{k_y(t)}{2\pi}\right)
 \end{aligned}$$

where $F(u, v)$ is the CSFT of $f(x, y)$

K-Space Interpretation of Demodulated Signal

- RF signal from the complete slice is given by

$$F \left(-\frac{k_x(t)}{2\pi}, -\frac{k_y(t)}{2\pi} \right) = R(t)e^{-j\omega_0 t}$$

where

$$k_x(t) = \int_0^t LG_x(\tau) d\tau$$

$$k_y(t) = \int_0^t LG_y(\tau) d\tau$$

- Strategy
 - Scan spatial frequencies by varying $k_x(t)$ and $k_y(t)$
 - Reconstruct image by performing (inverse) CSFT
 - $G_x(t)$ and $G_y(t)$ control velocity through K-space

Controlling K-Space Trajectory

- Relationship between gradient coil voltage and K-space

$$L_x \frac{di(t)}{dt} = v_x(t) \quad G_x(t) = M_x i(t)$$

$$L_y \frac{di(t)}{dt} = v_y(t) \quad G_y(t) = M_y i(t)$$

using this results in

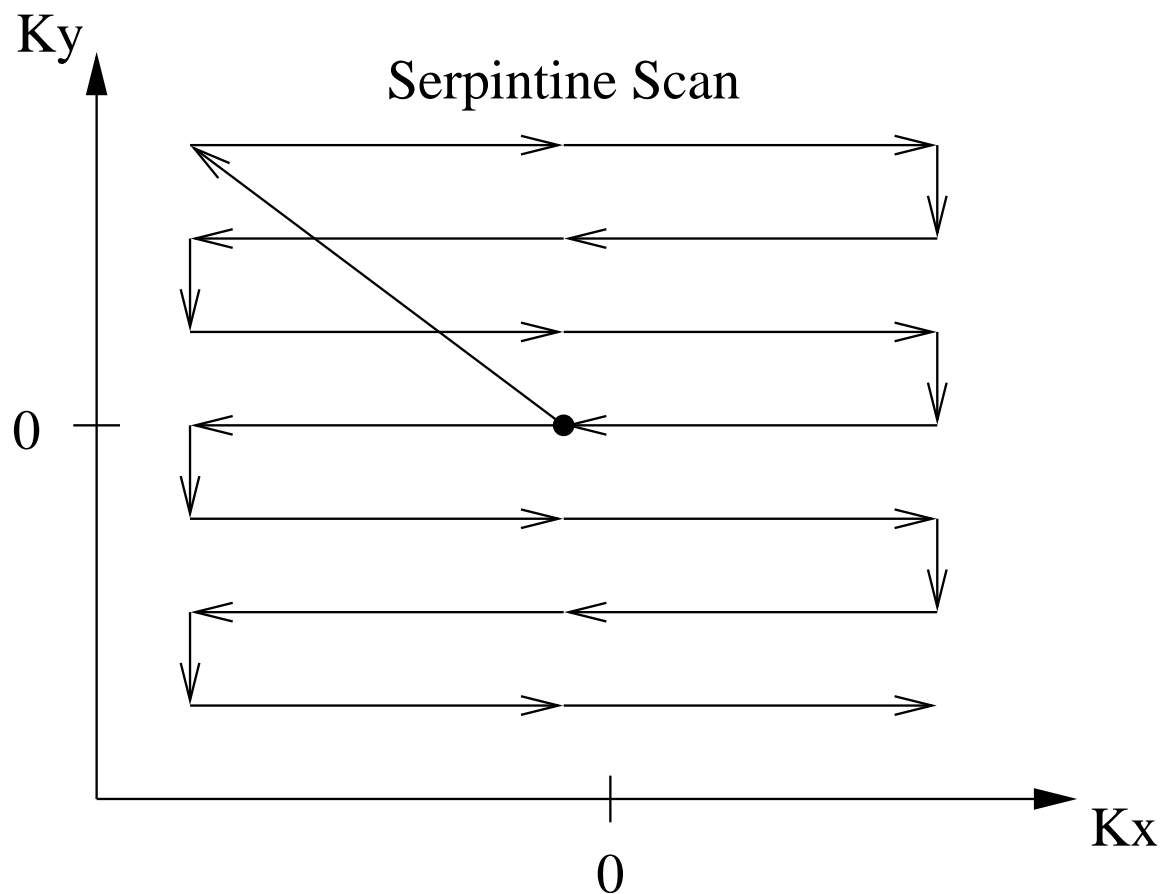
$$k_x(t) = \frac{LM_x}{L_x} \int_0^t \int_0^\tau v_x(s) ds d\tau$$

$$k_y(t) = \frac{LM_y}{L_y} \int_0^t \int_0^\tau v_y(s) ds d\tau$$

- $v_x(t)$ and $v_y(t)$ are like the accelerator peddles for $k_x(t)$ and $k_y(t)$

Echo Planer Imaging (EPI) Scan Pattern

- A commonly used raster scan pattern through K-space

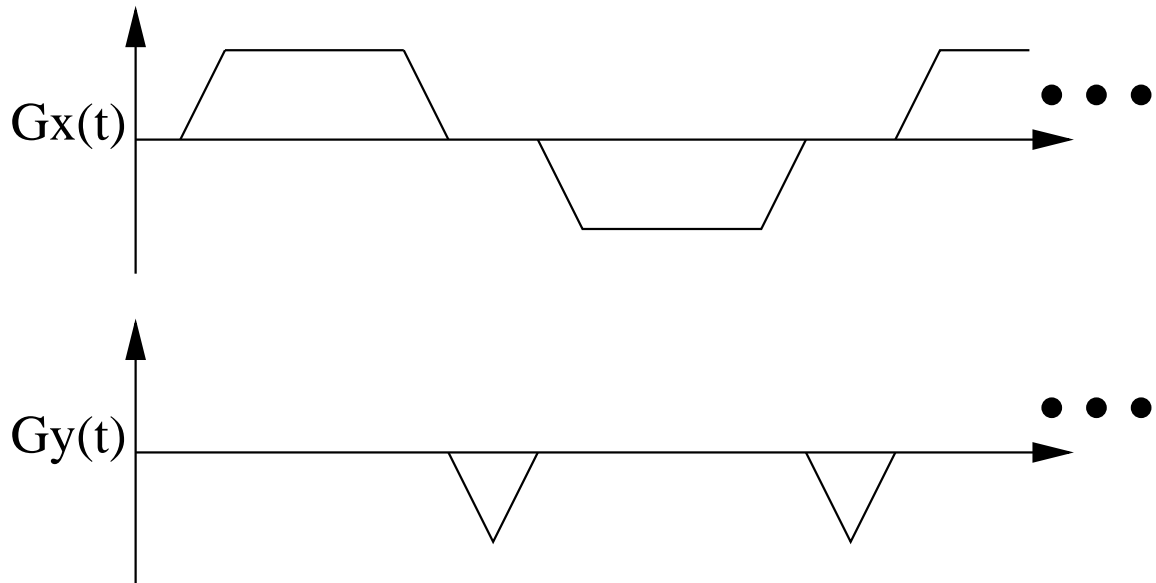


$$k_x(t) = L \int_0^t G_x(\tau) d\tau = \frac{LM_x}{L_x} \int_0^t \int_0^\tau v_x(s) ds d\tau$$

$$k_y(t) = L \int_0^t G_y(\tau) d\tau = \frac{LM_y}{L_y} \int_0^t \int_0^\tau v_y(s) ds d\tau$$

Gradient Waveforms for EPI

- Gradient waveforms in x and y look like



- Voltage waveforms in x and y look like

