Magnetic Resonance Imaging (MRI)

- Can be very high resolution
- No radiation exposure
- Very flexible and programmable
- Tends to be expensive, noisy, slow
MRI Attributes

• Based on magnetic resonance effect in atomic species
• Does not require any ionizing radiation
• Numerous modalities
  – Conventional anatomical scans
  – Functional MRI (fMRI)
  – MRI Tagging
• Image formation
  – RF excitation of magnetic resonance modes
  – Magnetic field gradients modulate resonance frequency
  – Reconstruction computed with inverse Fourier transform
  – Fully programmable
  – Requires an enormous (and very expensive) superconducting magnet
Magnetic Resonance

Magnetic Field

• Atom will precess at the Larmor frequency

\[ \omega_o = LM \]

• Quantities of importance

  \( M \) - magnitude of ambient magnetic field
  \( \omega_o \) - frequency of procession (radians per second)
  \( L \) - Larmor constant. Depends on choice of atom
The MRI Magnet

- Large super-conducting magnet
  - Uniform field within bore
  - Very large static magnetic field of $M_o$
Magnetic Field Gradients

- Magnetic field magnitude at the location \((x, y, z)\) has the form

\[
M(x, y, z) = M_o + xG_x + yG_y + zG_z
\]

- \(G_x, G_y, \) and \(G_z\) control magnetic field gradients
- Gradients can be changed with time
- Gradients are small compared to \(M_o\)

- For time varying gradients

\[
M(x, y, z, t) = M_o + xG_x(t) + yG_y(t) + zG_z(t)
\]
• Design RF pulse to excite protons in single slice
  – Turn off $x$ and $y$ gradients, i.e. $G_x = G_y = 0$.
  – Set $z$ gradient to fix positive value, $G_z > 0$.
  – Use the fact that resonance frequency is given by
    \[ \omega = L \left( M_o + zG_z \right). \]
Slice Select Pulse Design

• Design parameters
  – Slice center = \( z_c \).
  – Slice thickness = \( \Delta z \).
• Slice centered at \( z_c \) ⇒ pulse center frequency
  \[ f_c = \frac{LM_o}{2\pi} + \frac{z_cLG_z}{2\pi} = f_o + \frac{z_cLG_z}{2\pi}. \]
• Slice thickness \( \Delta z \) ⇒ pulse bandwidth
  \[ \Delta f = \frac{\Delta zLG_z}{2\pi}. \]
• Using these parameters, the pulse is given by
  \[ s(t) = e^{j2\pi f_c t} \text{sinc}(t\Delta f) \]
  and its CTFT is given by
  \[ S(f) = \text{rect}\left(\frac{(f - f_c)}{\Delta f}\right) \]
How Do We Imaging Selected Slice?

- Precessing atoms radiate electromagnetic energy at RF frequencies
- Strategy
  - Vary magnetic gradients along $x$ and $y$ axes
  - Measure received RF signal
  - Reconstruct image from RF measurements
Signal from a Single Voxel

- RF signal from a single voxel has the form
  \[ r(x, y, t) = f(x, y) e^{j\phi(t)} \]

  \( f(x, y) \) voxel dependent weighting
  - Depends on properties of material in voxel
  - Quantity of interest
  - Typically “weighted” by T1, T2, or T2*

  \( \phi(t) \) phase of received signal
  - Can be modulated using \( G_x \) and \( G_y \) magnetic field gradients
  - We assume that \( \phi(0) = 0 \)
Analysis of Phase

- Frequency = time derivative of phase

\[
\frac{d\phi(t)}{dt} = L M(x, y, t)
\]

\[
\phi(t) = \int_0^t L M(x, y, \tau) d\tau
\]

\[
= \int_0^t LM_o + xLG_x(\tau) + yLG_y(\tau) d\tau
\]

\[
= \omega_o t + xk_x(t) + yk_y(t)
\]

where we define

\[
\omega_o = L M_o
\]

\[
k_x(t) = \int_0^t LG_x(\tau) d\tau
\]

\[
k_y(t) = \int_0^t LG_y(\tau) d\tau
\]
Received Signal from Voxel

- RF signal from a single voxel has the form

\[
    r(t) = f(x, y) e^{j \phi(t)}
    \]

\[
    = f(x, y) e^{j \left(\omega_0 t + xk_x(t) + yk_y(t)\right)}
    \]

\[
    = f(x, y) e^{j \omega_0 t} e^{j \left(xk_x(t) + yk_y(t)\right)}
    \]
Received Signal from Selected Slice

- RF signal from the complete slice is given by

\[ R(t) = \int_{\mathbb{R}} \int_{\mathbb{R}} r(x, y, t) \, dx \, dy \]

\[ = \int_{\mathbb{R}} \int_{\mathbb{R}} f(x, y) e^{j\omega_0 t} e^{j(xk_x(t) + yk_y(t))} \, dx \, dy \]

\[ = e^{j\omega_0 t} \int_{\mathbb{R}} \int_{\mathbb{R}} f(x, y) e^{j(xk_x(t) + yk_y(t))} \, dx \, dy \]

\[ = e^{j\omega_0 t} F(-k_x(t), -k_y(t)) \]

were \( F(u, v) \) is the CSFT of \( f(x, y) \)
K-Space Interpretation of Demodulated Signal

• RF signal from the complete slice is given by

\[ F(-k_x(t), -k_y(t)) = R(t)e^{-j\omega_0 t} \]

where

\[ k_x(t) = \int_0^t L G_x(\tau) d\tau \]

\[ k_y(t) = \int_0^t L G_y(\tau) d\tau \]

• Strategy

  – Scan spatial frequencies by varying \( k_x(t) \) and \( k_y(t) \)
  – Reconstruct image by performing (inverse) CSFT
  – \( G_x(t) \) and \( G_y(t) \) control velocity through K-space
Controlling K-Space Trajectory

• Relationship between gradient coil voltage and K-space

\[
L_x \frac{d i(t)}{d t} = v_x(t) \quad G_x(t) = M_x i(t)
\]

\[
L_y \frac{d i(t)}{d t} = v_y(t) \quad G_y(t) = M_y i(t)
\]

using this results in

\[
k_x(t) = \frac{L M_x}{L_x} \int_0^t \int_0^\tau v_x(s) ds d\tau
\]

\[
k_y(t) = \frac{L M_y}{L_y} \int_0^t \int_0^\tau v_y(s) ds d\tau
\]

• \(v_x(t)\) and \(v_y(t)\) are like the accelerator peddles for \(k_x(t)\) and \(k_y(t)\)
Echo Planer Imaging (EPI) Scan Pattern

- A commonly used raster scan pattern through K-space

\[ k_x(t) = L \int_0^t G_x(\tau) d\tau = \frac{LM_x}{L_x} \int_0^t \int_0^\tau v_x(s) ds d\tau \]

\[ k_y(t) = L \int_0^t G_y(\tau) d\tau = \frac{LM_y}{L_y} \int_0^t \int_0^\tau v_y(s) ds d\tau \]
Gradient Waveforms for EPI

- Gradient waveforms in $x$ and $y$ look like

\[ G_x(t) \]
\[ G_y(t) \]

- Voltage waveforms in $x$ and $y$ look like

\[ V_x(t) \]
\[ V_y(t) \]