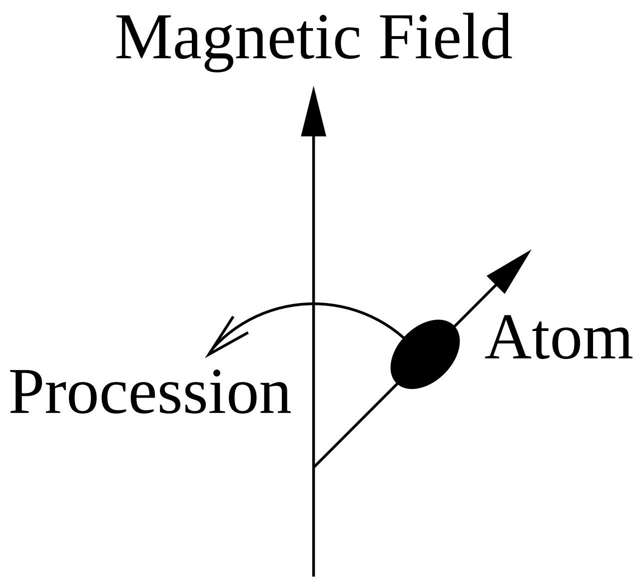
Magnetic Resonance Imaging (MRI)

- Can be very high resolution
- No radiation exposure
- Very flexible and programable
- Tends to be expensive, noisy, slow

MRI Attributes

- Based on magnetic resonance effect in atomic species
- Does not require any ionizing radiation
- Numerous modalities
  - Conventional anatomical scans
  - Functional MRI (fMRI)
  - MRI Tagging
- Image formation
  - RF excitation of magnetic resonance modes
  - Magnetic field gradients modulate resonance frequency
  - Reconstruction computed with inverse Fourier transform
  - Fully programmable
  - Requires an enormous (and very expensive) superconducting magnet

Magnetic Resonance



- Atom will precess at the Larmor frequency:

$$\omega_o = LM$$

- Quantities of importance:

  M - magnitude of ambient magnetic field

  ω_o - frequency of procession (radians per second)

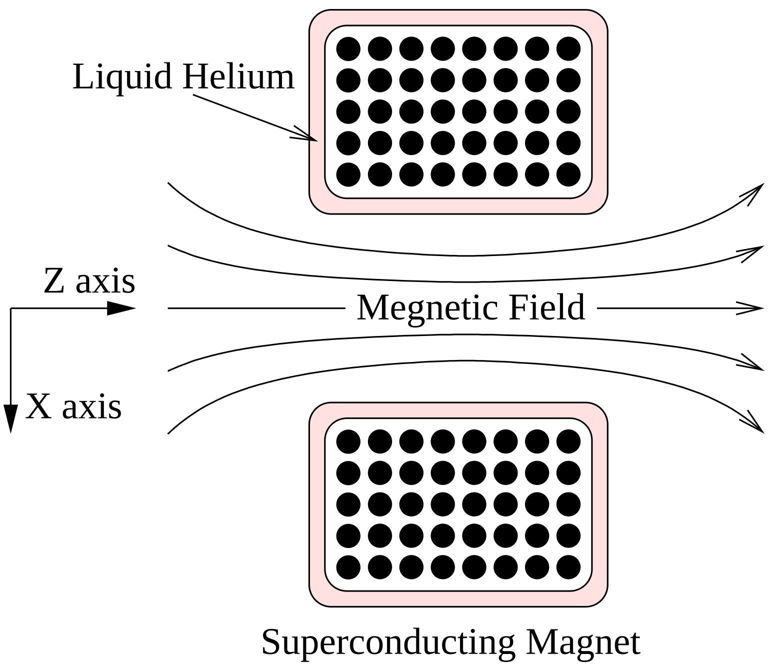
  L - Larmor constant. Depends on choice of atom

- Typical values:

  For hydrogen: 42.58 MHz/T; 1.5 T $\Rightarrow$ 63.87 MHz; 3 T $\Rightarrow$ 127.74 MHz

  For carbon 13: 10.705 MHz/T; 1.5 T $\Rightarrow$ 16.06 MHz; 3 T $\Rightarrow$ 32.12 MHz

The MRI Magnet



- Large super-conducting magnet
  - Uniform field within bore
  - Very large static magnetic field of M_o

Magnetic Field Gradients

- Magnetic field **magnitude** at the location (x, y, z) has the form

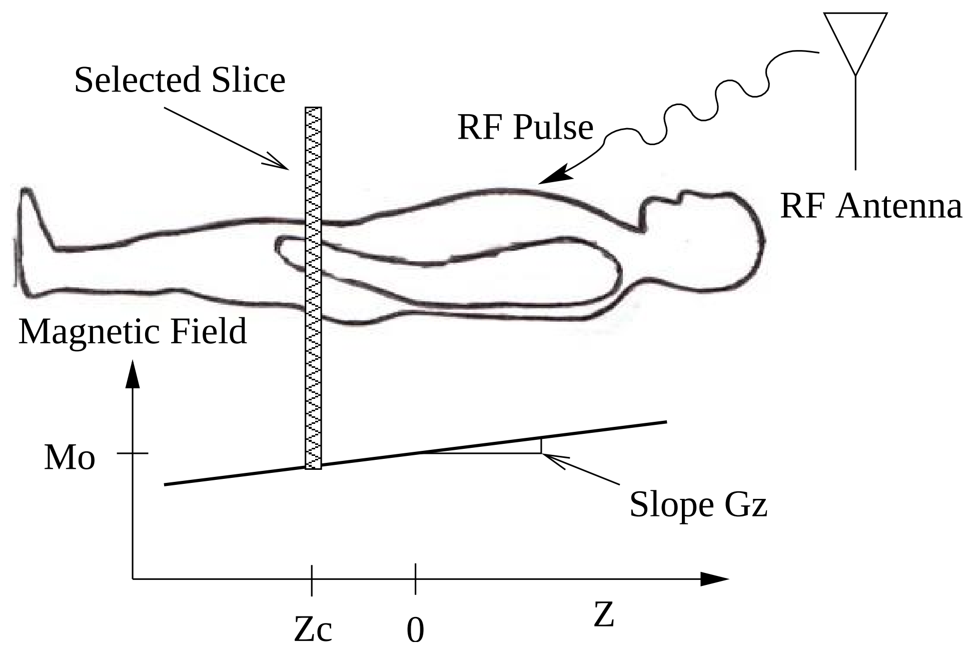
$$M(x, y, z) = M_o + xG_x + yG_y + zG_z$$

  - G_x, G_y, and G_z control magnetic field gradients
  - Gradients can be changed with time
  - Gradients are small compared to M_o

- For time varying gradients

$$M(x, y, z, t) = M_o + xG_x(t) + yG_y(t) + zG_z(t)$$

MRI Slice Select



- Design RF pulse to excite protons in single slice
  - Turn off x and y gradients, i.e. $G_x = G_y = 0$.
  - Set z gradient to fix positive value, $G_z > 0$.
  - Use the fact that resonance frequency is given by

$$\omega = L\left(M_o + zG_z\right) \ .$$

Slice Select Pulse Design

- Design parameters
  - Slice center = z_c.
  - Slice thickness = Δz.
- Slice centered at $z_c \Rightarrow$ pulse center frequency

$$f_c = \frac{LM_o}{2\pi} + \frac{z_c LG_z}{2\pi} = f_o + \frac{z_c LG_z}{2\pi} \ .$$

- Slice thickness $\Delta z \Rightarrow$ pulse bandwidth

$$\Delta f = \frac{\Delta z LG_z}{2\pi} \ .$$

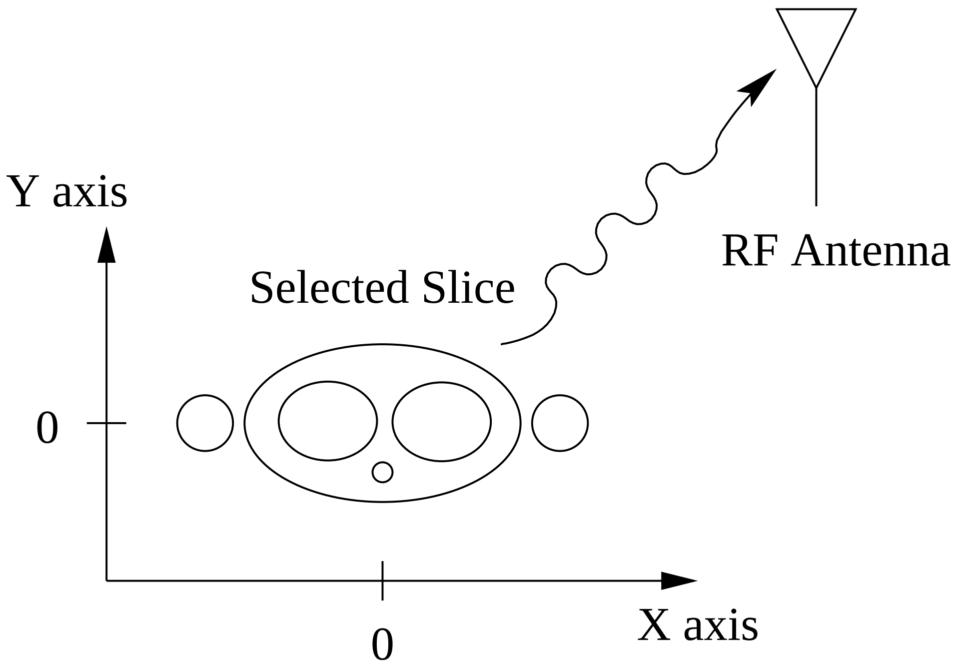
- Using these parameters, the pulse is given by

$$s(t) = e^{j2\pi f_c t}\text{sinc}\left(t\Delta f\right)$$

and its CTFT is given by

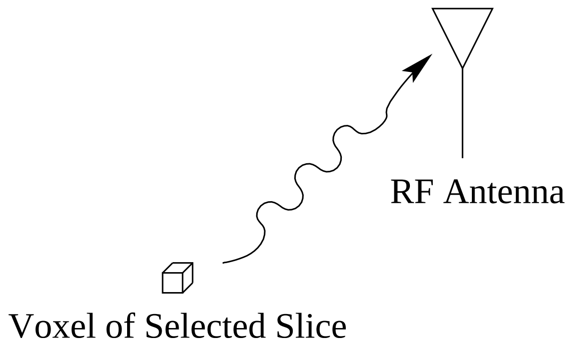
$$S(f) = \text{rect}\left(\frac{(f - f_c)}{\Delta f}\right)$$

How Do We Imaging Selected Slice?



- Precessing atoms radiate electromagnetic energy at RF frequencies
- Strategy
  - Vary magnetic gradients along x and y axies
  - Measure received RF signal
  - Reconstruct image from RF measurements

Signal from a Single Voxel



- RF signal from a single voxel has the form

$$r(x, y, t) = f(x, y)e^{j\phi(t)}$$

$f(x, y)$ voxel dependent weighting

- Depends on properties of material in voxel
- Quantity of interest
- Typically "weighted" by T1, T2, or T2*

$\phi(t)$ phase of received signal

- Can be modulated using G_x and G_y magnetic field gradients
- We assume that $\phi(0) = 0$

Analysis of Phase

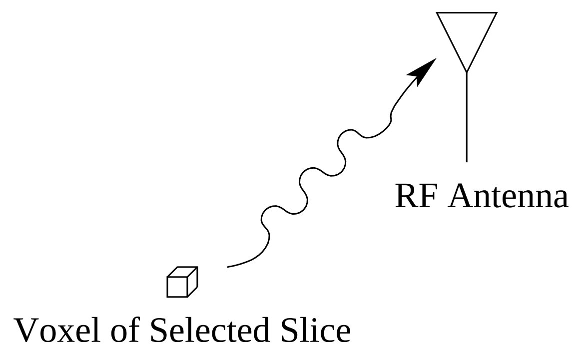
- Frequency = time derivative of phase

$$
\begin{aligned}
\frac{d\phi(t)}{dt} &= L\,M(x,y,t) \\
\phi(t) &= \int_0^t L\,M(x,y,\tau)d\tau \\
&= \int_0^t LM_o + xLG_x(\tau) + yLG_y(\tau)d\tau \\
&= \omega_o t + xk_x(t) + yk_y(t)
\end{aligned}
$$

where we define

$$
\begin{aligned}
\omega_o &= L\,M_o \\
k_x(t) &= \int_0^t LG_x(\tau)d\tau \\
k_y(t) &= \int_0^t LG_y(\tau)d\tau
\end{aligned}
$$

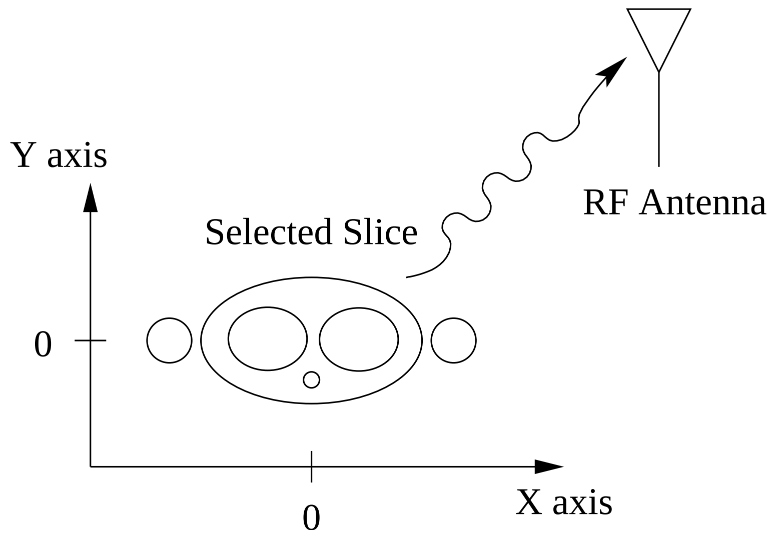
Received Signal from Voxel



- RF signal from a single voxel has the form

$$
\begin{aligned}
r(t) &= f(x,y)e^{j\phi(t)} \\
&= f(x,y)e^{j\left(\omega_o t + x k_x(t) + y k_y(t)\right)} \\
&= f(x,y)e^{j\omega_o t}e^{j\left(x k_x(t) + y k_y(t)\right)}
\end{aligned}
$$

Received Signal from Selected Slice



- RF signal from the complete slice is given by

$$
\begin{aligned}
R(t) &= \int_{\mathbb{R}} \int_{\mathbb{R}} r(x, y, t) dx dy \\
&= \int_{\mathbb{R}} \int_{\mathbb{R}} f(x, y) e^{j\omega_o t} e^{j\left(x k_x(t) + y k_y(t)\right)} dx dy \\
&= e^{j\omega_o t} \int_{\mathbb{R}} \int_{\mathbb{R}} f(x, y) e^{j\left(x k_x(t) + y k_y(t)\right)} dx dy \\
&= e^{j\omega_o t} F\left(-\frac{k_x(t)}{2\pi}, -\frac{k_y(t)}{2\pi}\right)
\end{aligned}
$$

were $F(u, v)$ is the CSFT of $f(x, y)$

K-Space Interpretation of Demodulated Signal

- RF signal from the complete slice is given by

$$F\left(-\frac{k_x(t)}{2\pi}, -\frac{k_y(t)}{2\pi}\right) = R(t)e^{-j\omega_o t}$$

where

$$\begin{aligned} k_x(t) &= \int_0^t LG_x(\tau)d\tau \\ k_y(t) &= \int_0^t LG_y(\tau)d\tau \end{aligned}$$

- Strategy
  - Scan spatial frequencies by varying $k_x(t)$ and $k_y(t)$
  - Reconstruct image by performing (inverse) CSFT
  - $G_x(t)$ and $G_y(t)$ control velocity through K-space

Controlling K-Space Trajectory

- Relationship between gradient coil voltage and K-space

$$L_x \frac{di(t)}{dt} = v_x(t) \quad G_x(t) = M_x i(t)$$

$$L_y \frac{di(t)}{dt} = v_y(t) \quad G_y(t) = M_y i(t)$$

using this results in

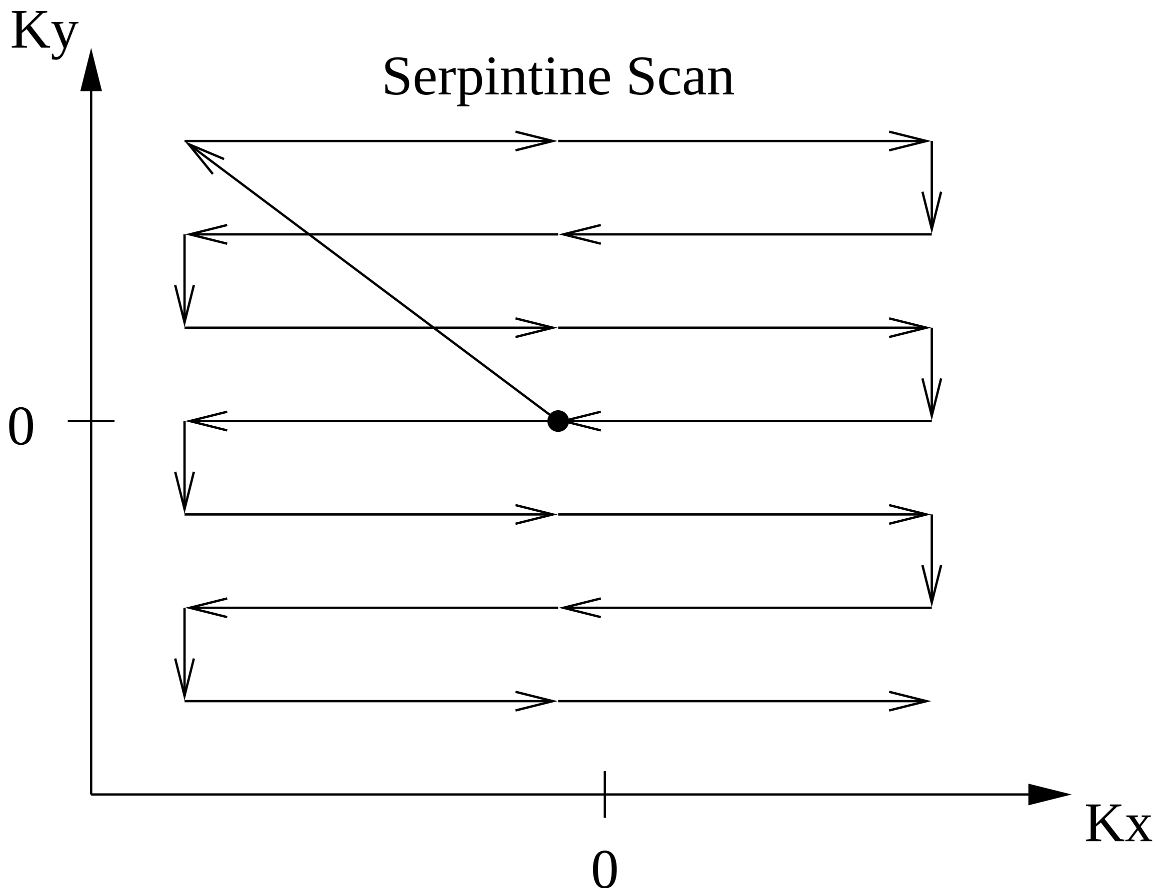
$$k_x(t) = \frac{LM_x}{L_x} \int_0^t \int_0^\tau v_x(s) ds d\tau$$

$$k_y(t) = \frac{LM_y}{L_y} \int_0^t \int_0^\tau v_y(s) ds d\tau$$

- $v_x(t)$ and $v_y(t)$ are like the accelerator peddles for $k_x(t)$ and $k_y(t)$

Echo Planer Imaging (EPI) Scan Pattern

- A commonly used raster scan pattern through K-space

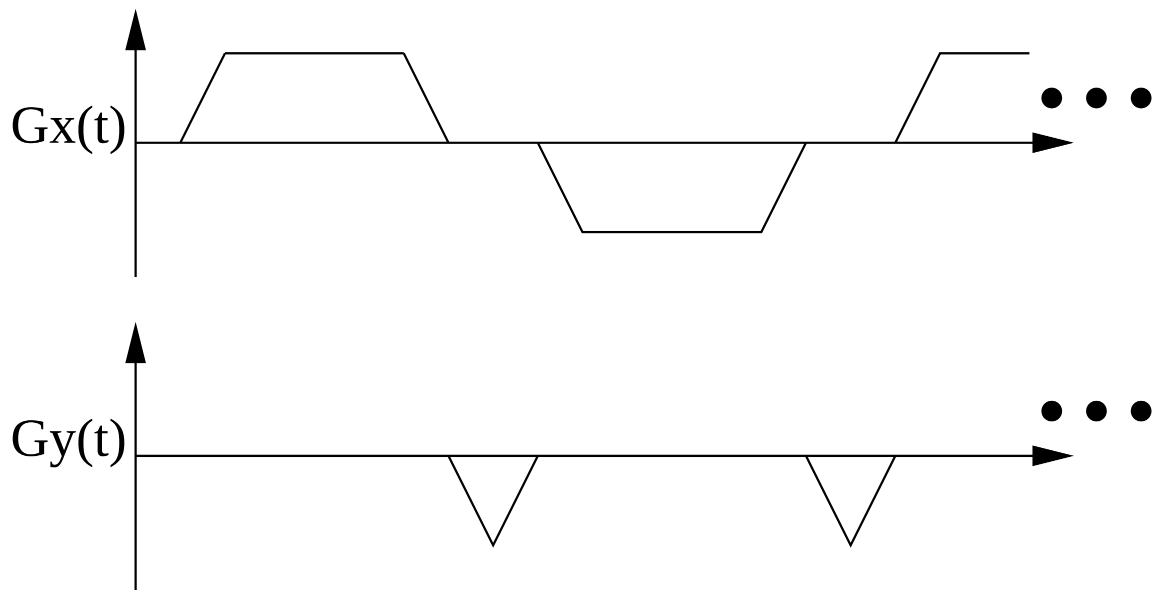


$$k_x(t) \;=\; L \int_0^t G_x(\tau)d\tau = \frac{LM_x}{L_x} \int_0^t \int_0^\tau v_x(s)dsd\tau$$

$$k_y(t) \;=\; L \int_0^t G_y(\tau)d\tau = \frac{LM_y}{L_y} \int_0^t \int_0^\tau v_y(s)dsd\tau$$

Gradient Waveforms for EPI

- Gradient waveforms in x and y look like



- Voltage waveforms in x and y look like