Magnetic Resonance Imaging (MRI)

- Can be very high resolution
- No radiation exposure
- Very flexible and programable
- Tends to be expensive, noisy, slow
MRI Attributes

- Based on magnetic resonance effect in atomic species
- Does not require any ionizing radiation
- Numerous modalities
  - Conventional anatomical scans
  - Functional MRI (fMRI)
  - MRI Tagging
- Image formation
  - RF excitation of magnetic resonance modes
  - Magnetic field gradients modulate resonance frequency
  - Reconstruction computed with inverse Fourier transform
  - Fully programmable
  - Requires an enormous (and very expensive) superconducting magnet
Magnetic Resonance

Atom will precess at the Larmor frequency

\[ \omega_o = LM \]

Quantities of importance

- $M$ - magnitude of ambient magnetic field
- $\omega_o$ - frequency of procession (radians per second)
- $L$ - Larmor constant. Depends on choice of atom
The MRI Magnet

- Large super-conducting magnet
  - Uniform field within bore
  - Very large static magnetic field of $M_o$
Magnetic Field Gradients

• Magnetic field **magnitude** at the location \((x, y, z)\) has the form

\[
M(x, y, z) = M_o + xG_x + yG_y + zG_z
\]

– \(G_x\), \(G_y\), and \(G_z\) control magnetic field gradients
– Gradients can be changed with time
– Gradients are small compared to \(M_o\)

• For time varying gradients

\[
M(x, y, z, t) = M_o + xG_x(t) + yG_y(t) + zG_z(t)
\]
• Design RF pulse to excite protons in single slice
  – Turn off $x$ and $y$ gradients, i.e. $G_x = G_y = 0$.
  – Set $z$ gradient to fix positive value, $G_z > 0$.
  – Use the fact that resonance frequency is given by
    \[ \omega = L \left( M_0 + zG_z \right). \]
Slice Select Pulse Design

- Design parameters
  - Slice center = $z_c$.
  - Slice thickness = $\Delta z$.

- Slice centered at $z_c$ $\Rightarrow$ pulse center frequency
  \[ f_c = \frac{LM_o}{2\pi} + \frac{z_c LG_z}{2\pi} = f_o + \frac{z_c LG_z}{2\pi}. \]

- Slice thickness $\Delta z$ $\Rightarrow$ pulse bandwidth
  \[ \Delta f = \frac{\Delta z LG_z}{2\pi}. \]

- Using these parameters, the pulse is given by
  \[ s(t) = e^{j2\pi f_c t} \text{sinc} (t\Delta f) \]
  and its CTFT is given by
  \[ S(f) = \text{rect} \left( \frac{(f - f_c)}{\Delta f} \right) \]
How Do We Imaging Selected Slice?

- Precessing atoms radiate electromagnetic energy at RF frequencies

- Strategy
  - Vary magnetic gradients along $x$ and $y$ axies
  - Measure received RF signal
  - Reconstruct image from RF measurements
Signal from a Single Voxel

- RF signal from a single voxel has the form
  \[ r(x, y, t) = f(x, y)e^{j\phi(t)} \]

  \( f(x, y) \) voxel dependent weighting
  - Depends on properties of material in voxel
  - Quantity of interest
  - Typically “weighted” by T1, T2, or T2*

  \( \phi(t) \) phase of received signal
  - Can be modulated using \( G_x \) and \( G_y \) magnetic field gradients
  - We assume that \( \phi(0) = 0 \)
Analysis of Phase

- Frequency = time derivative of phase

\[
\frac{d\phi(t)}{dt} = L\, M(x, y, t)
\]

\[
\phi(t) = \int_{0}^{t} L\, M(x, y, \tau)\, d\tau
\]

\[
= \int_{0}^{t} L\, M_{o} + x\, L\, G_{x}(\tau) + y\, L\, G_{y}(\tau)\, d\tau
\]

\[
= \omega_{o}t + x\, k_{x}(t) + y\, k_{y}(t)
\]

where we define

\[
\omega_{o} = L\, M_{o}
\]

\[
k_{x}(t) = \int_{0}^{t} L\, G_{x}(\tau)\, d\tau
\]

\[
k_{y}(t) = \int_{0}^{t} L\, G_{y}(\tau)\, d\tau
\]
• RF signal from a single voxel has the form

\[
    r(t) = f(x, y)e^{j\phi(t)} = f(x, y)e^{j(\omega_0 t + xk_x(t) + yk_y(t))} = f(x, y)e^{j\omega_0 t}e^{j(xk_x(t) + yk_y(t))}
\]
Received Signal from Selected Slice

RF signal from the complete slice is given by

\[ R(t) = \int_{\mathbb{R}} \int_{\mathbb{R}} r(x, y, t) \, dx \, dy \]

\[ = \int_{\mathbb{R}} \int_{\mathbb{R}} f(x, y) e^{j \omega_0 t} e^{j (x k_x(t) + y k_y(t))} \, dx \, dy \]

\[ = e^{j \omega_0 t} \int_{\mathbb{R}} \int_{\mathbb{R}} f(x, y) e^{j (x k_x(t) + y k_y(t))} \, dx \, dy \]

\[ = e^{j \omega_0 t} F(-k_x(t), -k_y(t)) \]

were \( F(u, v) \) is the CSFT of \( f(x, y) \)
K-Space Interpretation of Demodulated Signal

- RF signal from the complete slice is given by
  \[ F(-k_x(t), -k_y(t)) = R(t)e^{-j\omega_0 t} \]

  where

  \[ k_x(t) = \int_0^t LG_x(\tau) d\tau \]

  and

  \[ k_y(t) = \int_0^t LG_y(\tau) d\tau \]

- Strategy
  - Scan spatial frequencies by varying \( k_x(t) \) and \( k_y(t) \)
  - Reconstruct image by performing (inverse) CSFT
  - \( G_x(t) \) and \( G_y(t) \) control velocity through K-space
Controlling K-Space Trajectory

- Relationship between gradient coil voltage and K-space

\[
L_x \frac{di_x(t)}{dt} = v_x(t) \quad G_x(t) = M_x i(t)
\]

\[
L_y \frac{di_y(t)}{dt} = v_y(t) \quad G_y(t) = M_y i(t)
\]

using this results in

\[
k_x(t) = \frac{LM_x}{L_x} \int_0^t \int_0^\tau v_x(s) ds d\tau
\]

\[
k_y(t) = \frac{LM_y}{L_y} \int_0^t \int_0^\tau v_y(s) ds d\tau
\]

- \(v_x(t)\) and \(v_y(t)\) are like the accelerator peddles for \(k_x(t)\) and \(k_y(t)\)
Echo Planer Imaging (EPI) Scan Pattern

- A commonly used raster scan pattern through K-space

$$k_x(t) = L \int_0^t G_x(\tau) \, d\tau = \frac{LM_x}{L_x} \int_0^t \int_0^\tau v_x(s) \, ds \, d\tau$$

$$k_y(t) = L \int_0^t G_y(\tau) \, d\tau = \frac{LM_y}{L_y} \int_0^t \int_0^\tau v_y(s) \, ds \, d\tau$$
Gradient Waveforms for EPI

- Gradient waveforms in $x$ and $y$ look like

\[ G_x(t) \]

\[ G_y(t) \]

- Voltage waveforms in $x$ and $y$ look like

\[ V_x(t) \]

\[ V_y(t) \]