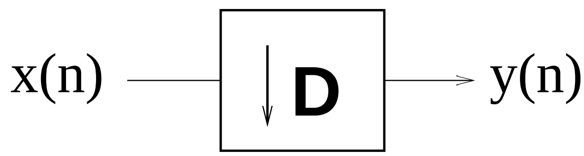
1-D Rate Conversion

- Decimation
  - Reduce the sampling rate of a discrete-time signal.
  - Low sampling rate reduces storage and computation requirements.
- Interpolation
  - Increase the sampling rate of a discrete-time signal.
  - Higher sampling rate preserves fidelity.

1-D Periodic Subsampling

- Time domain subsampling of $x(n)$ with period D

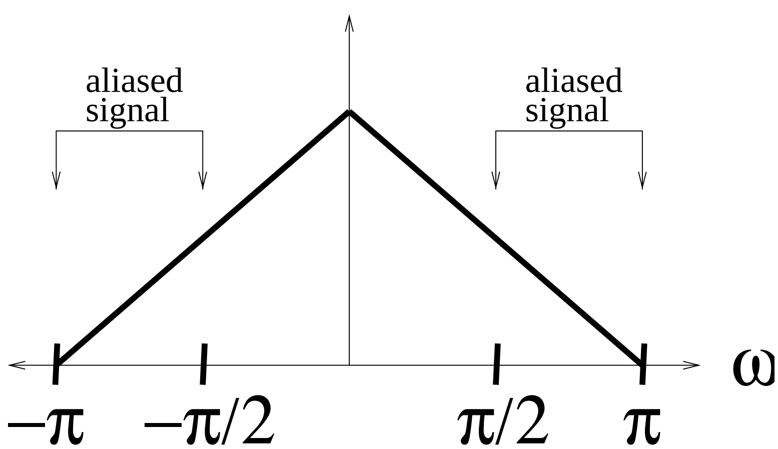
$$y(n) = x(Dn)$$



- Frequency domain representation

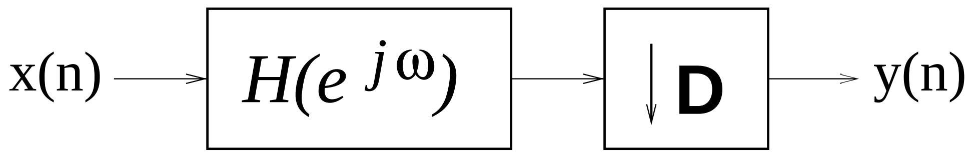
$$Y(e^{j\omega}) = \frac{1}{L} \sum_{k=-\infty}^{\infty} X\left(e^{j(\omega-2\pi k)/L}\right)$$

- Problem: Frequencies above π/L will alias.
- Example when $L = 2$



- Solution: Filter out frequencies below π/L.

Decimation System



- Apply the filter $H(e^{j\omega})$ to remove high frequencies
  - For $|\omega| < \pi$

$$H(e^{j\omega}) = \text{rect}\left(L\omega/\pi\right)$$

  - For all ω

$$\begin{aligned} H(e^{j\omega}) &= \sum_{k=-\infty}^{\infty} \text{rect}\left(L\frac{\omega - k2\pi}{\pi}\right) \\ &= \text{prect}_{\pi/L}(\omega) \end{aligned}$$

  - Impulse response

$$h(n) = \frac{1}{L}\text{sinc}(n/L)$$

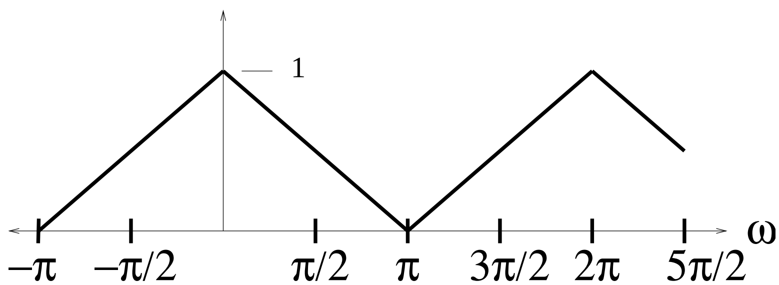
- Frequency domain representation

$$Y(e^{j\omega}) = \frac{1}{L}\sum_{k=-\infty}^{\infty} H\left(e^{j(\omega-2\pi k)/L}\right) X\left(e^{j(\omega-2\pi k)/L}\right)$$

Graphical View of Decimation

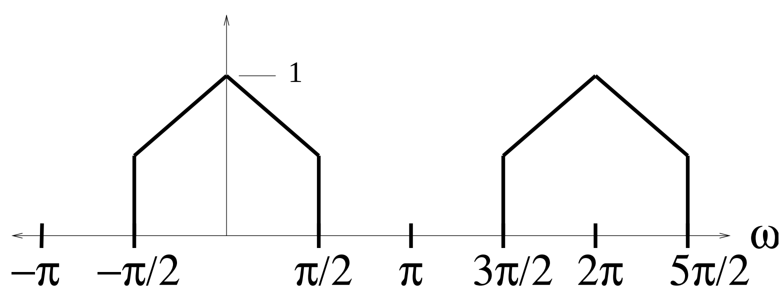
- Spectral content of signal

$$X\left(e^{j\omega}\right)$$



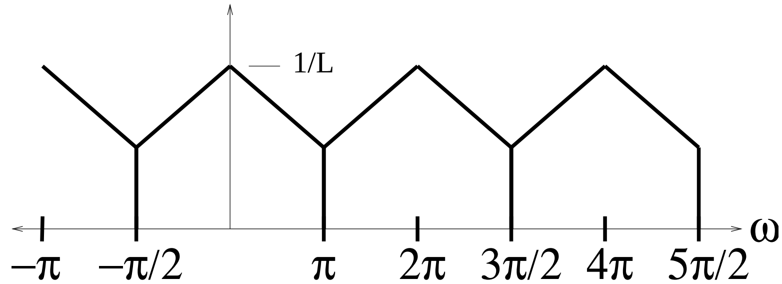
- Spectral content of filtered signal

$$H\left(e^{j\omega}\right)X\left(e^{j\omega}\right)$$



- Spectral content of decimated signal

$$Y(e^{j\omega}) = \frac{1}{L}\sum_{k=-\infty}^{\infty} H\left(e^{j(\omega-2\pi k)/L}\right)X\left(e^{j(\omega-2\pi k)/L}\right)$$



Decimation for Images

- Extension to decimation of images is direct
- Apply 2-D Filter

$$f(i,j) = h(i,j) * x(i,j)$$

- Subsample result

$$y(i,j) = f(Li, Lj)$$

- Ideal choice of filter is

$$h(m,n) = \frac{1}{L^2}\text{sinc}(m/L)\text{sinc}(n/L)$$

- Problems:
  - Filter has infinit extent.
  - Filter is not strictly positive.

Alternative Filters for Image Decimation

- Direct subsampling

$$h(m,n) = \delta(m,n)$$

- Advantages/Disadvantages:
  - Low computation
  - Excessive aliasing
- Block averaging

$$h(m,n) = \delta(m,n)+\delta(m+1,n)+\delta(m,n+1)+\delta(m+1,n+1)$$

- Advantages/Disadvantages:
  - Low computation
  - Some aliasing