Edit Assignment

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QUESTION 1

Title

POINTS

2

Delete Question

PROBLEM

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```
**Rules:** I understand that this is an open book exam that
shall be done within the allotted time of 120 minutes. I can
use my notes, previous posted exams and exam solutions, and
web resources. However, I will not communicate with any
other person other than the official exam proctors during
the exam, and I will not seek or accept help from any other
persons other than the official proctors.

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![CheatSheet.jpg](/files/7c5036d9-76d4-4153-8f83-
d61ae3d9d1cb)
```

Add **Subquestion**

QUESTION 2

Unsharp Mask Filter

POINTS

36

Delete Question

DESCRIPTION

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```
Consider the following 2D system with input x(m, n) and
output y(m, n),
$$
y(m,n) = x(m,n) + \lambda \left\{ x(m,n) - h(m,n) * x(m,n)
\right\}
$$
where $$\lambda>0$$, and $$h(m,n) = g(m)\, g(n)$$ where
$$
g(n) = (1/4)\delta(n-1) + (1/2)\delta(n) + (1/4)\delta(n+1)\
```

.
$$

QUESTION 2.1

POINTS

✖ Delete Question

Title

6

PROBLEM

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Calculate $$H(e^{j\mu},e^{j\nu})$$, the DSFT of $$h(m,n)$$.

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QUESTION 2.2

POINTS

✖ Delete Question

Title

6

PROBLEM

Insert Images   Insert Field

Calculate a closed form expression for $$p(m,n)$$, the point spread function of the overall system where $$y(m,n)=p(m,n)*x(m,n)$$.

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QUESTION 2.3

POINTS

✖ Delete Question

Title

6

PROBLEM

Insert Images   Insert Field

Is $$p(m,n)$$ a separable function?
Justify your answer.

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QUESTION 2.4

POINTS

✖ Delete Question

Title

6

PROBLEM

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```
Calculate a closed form expression for
$$P(e^{j\mu},e^{j\nu})$$, the DSFT of $$p(m,n)$$.

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```

QUESTION 2.5

POINTS

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Title

6

PROBLEM

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```
Sketch the 2D function $$P(e^{j\mu},e^{j\nu})$$ for
$$(\mu,\nu)\in [-\pi,\pi]^2$$ for $$\lambda=1$$.

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```

QUESTION 2.6

POINTS

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Title

6

PROBLEM

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```
What is the value of $$P(e^{j\mu},e^{j\nu})$$ at
$$(\mu,\nu)=(0,0)$$?
Explain the meaning of this answer.

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```

↳Add **Question 2.7**

QUESTION 3

POINTS

✖ Delete Question

Tomography

34

DESCRIPTION

DESCRIPTION

Insert Images

Consider the 2D discrete space signal $$x(m,n)$$ with the DSFT of $$X(e^{j\mu }, e^{j\nu })$$ given by
$$
X(e^{j\mu }, e^{j\nu }) = \sum_{m=-\infty}^\infty \sum_{n=-\infty}^\infty x(m,n) e^{-j (m \mu + n\nu)} \ .
$$
Then define the vertical and horizontal projections given by
$$
p_v (n) = \sum_{m=-\infty}^\infty x(m,n)
$$
$$
p_h (m) = \sum_{n=-\infty}^\infty x(m,n)
$$
with corresponding DTFTs given by
$$
P_v (e^{j\omega }) = \sum_{n=-\infty}^\infty p_v (n) e^{-j n\omega }
$$
$$
P_h (e^{j\omega }) = \sum_{m=-\infty}^\infty p_h(m) e^{-j m\omega }
$$

QUESTION 3.1

POINTS

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Title

5

PROBLEM

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Derive an expression for $$P_v (e^{j\omega })$$ in terms of $$X(e^{j\mu }, e^{j\nu })$$.

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QUESTION 3.2

POINTS

Delete Question

Title

5

PROBLEM

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Derive an expression for $$P_h (e^{j\omega })$$ in terms of $$X(e^{j\mu }, e^{j\nu })$$.

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QUESTION 3.3

POINTS

✖ Delete Question

Title

8

PROBLEM

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```
Derive an expression for $$\sum_{n=-\infty}^\infty p_v
(n)$$ in terms of $$X( e^{j\mu }, e^{j\nu } )$$.

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```

QUESTION 3.4

POINTS

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Title

8

PROBLEM

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```
Assume that you are able to measure the projections $$p_v
(n)$$ and $$p_h (m)$$. Then what do these functions tell
you about the function $$X( e^{j\mu }, e^{j\nu } )$$?
Sketch a figure that illustrates the answer to this
problem.

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```

QUESTION 3.5

POINTS

✖ Delete Question

Title

8

PROBLEM

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```
Do the functions $$p_v (n)$$ and $$p_h (m)$$ together
contain sufficient information to reconstruction the
function $$x(m,n)$$?
If so, provide a reconstruction algorithm; if not,
provide a counter example.

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```

↳ Add **Question 3.6**

QUESTION 4

POINTS

✖ Delete Question

Sampling

32

DESCRIPTION

Insert Images

```
Let $$g(x,y) = {\tt sinc}(x/2,y/2)$$ and let $$s(m,n) =
g(mT,nT)$$ for some $$T>0$$.
```

QUESTION 4.1

POINTS

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Title

8

PROBLEM

Insert Images   Insert Field

```
Calculate $$G(u,v)$$ the CSFT of $$g(x,y)$$ and sketch it
on $$(u,v)\in [-2,2]^2$$.

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```

QUESTION 4.2

POINTS

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Title

8

PROBLEM

Insert Images   Insert Field

```
Calculate $$S(e^{j\mu},e^{j\nu})$$ the DSFT of
$$s(m,n)$$, and sketch it for $$T=1$$ on $$(μ,ν)\in
[-2\pi,2\pi]^2$$.

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```

QUESTION 4.3

POINTS

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Title 8

PROBLEM

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```
What value of $$T$$ corresponds to the Nyquist sampling
rate?

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```

QUESTION 4.4

POINTS

✖ Delete Question

Title

8

PROBLEM

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```
Calculate the sampled signal, $$s(m,n)$$, and its Fourier
transform, $$S(e^{j\mu},e^{j\nu})$$, when $$T=2$$.

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```

↳Add **Question 4.5**

+ Add **Question 5**

Save Assignment

Q1

2 Points

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Fact Sheet

- Function definitions

$$\text{rect}(t) \triangleq \begin{cases} 1 & \text{for } |t| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Lambda(t) \triangleq \begin{cases} 1 - |t| & \text{for } |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\text{sinc}(t) \triangleq \frac{\sin(\pi t)}{\pi t}$$

- CTFT

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df$$

- CTFT Properties

$$x(-t) \overset{CTFT}{\Leftrightarrow} X(-f)$$

$$x(t - t_0) \overset{CTFT}{\Leftrightarrow} X(f) e^{-j2\pi f t_0}$$

$$x(at) \overset{CTFT}{\Leftrightarrow} \frac{1}{|a|} X(f/a)$$

$$X(t) \overset{CTFT}{\Leftrightarrow} x(-f)$$

$$x(t) e^{j2\pi f_0 t} \overset{CTFT}{\Leftrightarrow} X(f - f_0)$$

$$x(t) y(t) \overset{CTFT}{\Leftrightarrow} X(f) * Y(f)$$

$$x(t) * y(t) \overset{CTFT}{\Leftrightarrow} X(f) Y(f)$$

$$\int_{-\infty}^{\infty} x(t) y^*(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(f) Y^*(f) df$$

- CTFT pairs

$$\text{sinc}(t) \overset{CTFT}{\Leftrightarrow} \text{rect}(f)$$

$$\text{rect}(t) \overset{CTFT}{\Leftrightarrow} \text{sinc}(f)$$

For $a > 0$

$$\frac{1}{(n-1)!} t^{n-1} e^{-at} u(t) \overset{CTFT}{\Leftrightarrow} \frac{1}{(j2\pi f + a)^n}$$

- CSFT

$$F(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{-j2\pi(ux+vy)} dx dy$$

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u, v) e^{j2\pi(ux+vy)} du dv$$

- DTFT

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

- DTFT pairs

$$a^n u(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{1 - a e^{-j\omega}}$$

$$(n+1) a^n u(n) \overset{DTFT}{\Leftrightarrow} \frac{1}{(1 - a e^{-j\omega})^2}$$

- Rep and Comb relations

$$\text{rep}_T[x(t)] = \sum_{k=-\infty}^{\infty} x(t - kT)$$

$$\text{comb}_T[x(t)] = x(t) \sum_{k=-\infty}^{\infty} \delta(t - kT)$$

$$\text{comb}_T[x(t)] \overset{CTFT}{\Leftrightarrow} \frac{1}{T} \text{rep}_{\frac{1}{T}}[X(f)]$$

$$\text{rep}_T[x(t)] \overset{CTFT}{\Leftrightarrow} \frac{1}{T} \text{comb}_{\frac{1}{T}}[X(f)]$$

- Sampling and Reconstruction

$$y(n) = x(nT)$$

$$Y(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right)$$

$$s(t) = \sum_{k=-\infty}^{\infty} y(k) \delta(t - kT)$$

$$S(e^{j\omega}) = Y\left(e^{j\omega T}\right)$$

Q2 Unsharp Mask Filter

36 Points

Consider the following 2D system with input x(m, n) and output y(m, n),

$$y(m,n) = x(m,n) + \lambda \{x(m,n) - h(m,n) * x(m,n)\}$$

where $\lambda > 0$, and $h(m,n) = g(m)\, g(n)$ where

$$g(n) = (1/4)\delta(n-1) + (1/2)\delta(n) + (1/4)\delta(n+1)\ .$$

Q2.1

6 Points

Calculate $H(e^{j\mu}, e^{j\nu})$, the DSFT of $h(m,n)$.

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Q2.2

6 Points

Calculate a closed form expression for $p(m,n)$, the point spread function of the overall system where $y(m,n) = p(m,n) * x(m,n)$.

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Q2.3

6 Points

Is $p(m,n)$ a separable function?
Justify your answer.

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Q2.4

6 Points

Calculate a closed form expression for $P(e^{j\mu}, e^{j\nu})$, the DSFT of $p(m,n)$.

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Q2.5

6 Points

Sketch the 2D function $P(e^{j\mu}, e^{j\nu})$ for $(\mu, \nu) \in [-\pi, \pi]^2$ for $\lambda = 1$.

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Q2.6

6 Points

What is the value of $P(e^{j\mu}, e^{j\nu})$ at $(\mu, \nu) = (0, 0)$?
Explain the meaning of this answer.

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Q3 Tomography

34 Points

Consider the 2D discrete space signal $x(m, n)$ with the DSFT of $X(e^{j\mu}, e^{j\nu})$ given by

$$X(e^{j\mu}, e^{j\nu}) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(m, n) e^{-j(m\mu + n\nu)} .$$

Then define the vertical and horizontal projections given by

$$p_v(n) = \sum_{m=-\infty}^{\infty} x(m, n)$$

$$p_h(m) = \sum_{n=-\infty}^{\infty} x(m, n)$$

with corresponding DTFTs given by

$$P_v(e^{j\omega}) = \sum_{n=-\infty}^{\infty} p_v(n) e^{-jn\omega}$$

$$P_h(e^{j\omega}) = \sum_{m=-\infty}^{\infty} p_h(m) e^{-jm\omega}$$

Q3.1
5 Points

Derive an expression for $P_v(e^{j\omega})$ in terms of $X(e^{j\mu}, e^{j\nu})$.

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Q3.2
5 Points

Derive an expression for $P_h(e^{j\omega})$ in terms of $X(e^{j\mu}, e^{j\nu})$.

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Q3.3
8 Points

Derive an expression for $\sum_{n=-\infty}^{\infty} p_v(n)$ in terms of $X(e^{j\mu}, e^{j\nu})$.

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Q3.4
8 Points

Assume that you are able to measure the projections $p_v(n)$ and $p_h(m)$. Then what do these functions tell you about the function $X(e^{j\mu}, e^{j\nu})$?
Sketch a figure that illustrates the answer to this problem.

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Q3.5
8 Points

Do the functions $p_v(n)$ and $p_h(m)$ together contain sufficient information to reconstruction the function $x(m, n)$?
If so, provide a reconstruction algorithm; if not, provide a counter example.

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Q4 Sampling
32 Points

Let $g(x, y) = \texttt{sinc}(x/2, y/2)$ and let $s(m, n) = g(mT, nT)$ for some $T > 0$.

Q4.1
8 Points

Calculate $G(u, v)$ the CSFT of $g(x, y)$ and sketch it on $(u, v) \in [-2, 2]^2$.

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Q4.2
8 Points

Calculate $S(e^{j\mu}, e^{j\nu})$ the DSFT of $s(m, n)$, and sketch it for $T = 1$ on $(\mu, \nu) \in [-2\pi, 2\pi]^2$.

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Q4.3
8 Points

What value of T corresponds to the Nyquist sampling rate?

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Q4.4
8 Points

Calculate the sampled signal, $s(m, n)$, and its Fourier transform, $S(e^{j\mu}, e^{j\nu})$, when $T = 2$.

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