Markov Random Fields and Stochastic Image Models

Charles A. Bouman
School of Electrical and Computer Engineering
Purdue University
Phone: (317) 494-0340
Fax: (317) 494-3358
email bouman@ecn.purdue.edu
Available from: http://dynamo.ecn.purdue.edu/~bouman/

Special thanks to:

Ken Sauer
Department of Electrical Engineering
University of Notre Dame

Suhail Saquib
School of Electrical and Computer Engineering
Purdue University

1 Breakdown of References in Statistical Image Modeling

1. Overview references [97, 86, 49, 53, 158, 4, 44]
2. Type of Random Field Model
   (a) Discrete Models
      i. Hidden Markov models [131, 132]
      ii. Markov Chains [41, 42, 152, 129]
      iii. Ising model [124, 123, 119, 127, 97, 128]
      iv. Discrete MRF [13, 14, 156, 47, 157, 46, 16, 165, 36, 50, 48, 113, 163, 96, 49, 70, 101, 153, 54, 177, 118, 120, 23, 88, 172, 89, 37, 122, 125, 137, 164, 94, 116, 11, 39, 75, 168, 90]
      v. MRF with Line Processes[66, 52, 173, 171, 174, 169, 167]
   (b) Continuous Models
      i. AR and Simultaneous AR [92, 91, 112]
      ii. Gaussian MRF [18, 15, 84, 92, 91, 33, 111, 149, 38, 103, 144]
      iii. Nonconvex potential functions [68, 69, 21, 79, 104, 64, 32, 140]
      iv. Convex potential functions [17, 73, 104, 105, 151, 24, 143, 87, 25, 27, 32, 146, 26, 145, 147]
3. Regularization approaches

   (a) Quadratic [161, 154, 134, 99, 100, 95, 135, 58]
   (b) Nonconvex [136, 82, 85, 155, 19, 20]
   (c) Convex [151, 3]

4. Simulation and Stochastic Optimization Methods [115, 78, 126, 97, 66, 138, 59, 74, 60, 61]

5. Computational Methods used with MRF Models
   (a) Simulation based estimators [113, 153, 54, 39, 26]
   (b) Discrete optimization
      i. Simulated annealing [66, 163, 54, 177]
      ii. Recursive optimization [47, 48, 165, 152, 88, 168, 169, 90]
      iii. Greedy optimization [156, 16, 157, 36, 50, 101, 153, 54, 122, 89]
      iv. Multiscale optimization [22, 70, 23, 125, 94, 102, 107]
      v. Mean field theory [172, 173, 171, 174, 167]
   (c) Continuous optimization
      i. Simulated annealing [149]
      ii. Gradient ascent [84, 146, 147]
      iii. Conjugate gradient [10]
      iv. EM [68, 69, 79, 104, 73, 80]
      v. ICM/Gauss-Seidel/ICD [24, 143, 144, 25, 27]
      vi. Continuation methods [19, 20, 149, 140]

6. Parameter Estimation
   (a) For MRF
      i. Discrete MRF
         A. Maximum likelihood [127, 62, 69, 128, 118, 105]
         B. Coding/maximum pseudolikelihood [15, 16, 18, 67, 101]
         C. Least squares [48, 75]
      ii. Continuous MRF
         A. Gaussian [92, 91, 33, 111, 38, 112, 103]
         B. Non-Gaussian [121, 145, 26, 130, 142, 141]
      iii. EM based [69, 172, 173, 39, 176, 26, 174, 130, 142, 141]
   (b) For other models
      i. EM algorithm for HMM's and mixture models [9, 8, 45, 166, 133, 1]
      ii. Order identification [2, 91, 139, 37, 175, 176]

7. Application

(a) Texture classification [92, 33, 38, 112]

(b) Texture modeling [55, 91]

(c) Segmentation remotely sensed imagery [156, 157, 96, 177, 137, 28, 89, 29]

(d) Segmentation of documents [153, 54]

(e) Segmentation (nonspecific) [47, 16, 46, 36, 48, 50, 163, 113, 93, 101, 111, 23, 37, 112, 122, 164, 94, 117, 11, 39, 107, 176, 168, 90]

(f) Boundary and edge detection [41, 42, 56, 152, 63, 171]

(g) Image restoration [84, 66, 165, 93, 149, 88, 87, 173, 80, 147]

(h) Image interpolation [146]

(i) Optical flow estimation [85, 98, 81, 108, 140, 174]

(j) Texture modeling [92, 91, 44, 33, 38, 120, 112, 109, 55, 110]

(k) Tomography [77, 68, 69, 79, 73, 104, 105, 24, 143, 25, 144, 32, 26, 27]

(l) Crystallography [52]

(m) Template matching [162, 150]

(n) Image interpretation [116]

8. Multiscale Bayesian Models

   (a) Discrete model [28, 29, 40, 150]

   (b) Continuous model [12, 34, 5, 6, 7, 109, 35, 51, 108, 110, 162]

   (c) Parameter estimation [34, 29, 162, 150]

9. Multigrid techniques [76, 30, 31, 114, 57]

References

[1] M. Aitkin and D. B. Rubin. Estimation and hypothesis testing in finite mixture models. *Journal of the Royal Statistical Society B*, 47(1):67–75, 1985.

[2] H. Akaike. A new look at the statistical model identification. *IEEE Trans. Automat. Contr.*, AC-19:716–723, December 1974.

[3] S. Alliney and S. A. Ruzinsky. An algorithm for the minimization of mixed l_1 and l_2 norms with application to Bayesian estimation. *IEEE Trans. on Signal Processing*, 42(3):618–627, March 1994.

[4] P. Barone, A. Frigessi, and M. Piccioni, editors. *Stochastic models, statistical methods, and algorithms in image analysis*. Springer-Verlag, Berlin, 1992.

[5] M. Basseville, A. Benveniste, K. Chou, S. Golden, R. Nikoukhah, and A. S. Willsky. Modeling and estimation of multiresolution stochastic processes. *IEEE Trans. on Information Theory*, 38(2):766–784, March 1992.

[6] M. Basseville, A. Benveniste, and A. Willsky. Multiscale autoregressive processes, part i: Schur-Levinson parametrizations. *IEEE Trans. on Signal Processing*, 40(8):1915–1934, 1992.

[7] M. Basseville, A. Benveniste, and A. Willsky. Multiscale autoregressive processes, part ii: Lattice structures for whitening and modeling. *IEEE Trans. on Signal Processing*, 40(8):1915–1934, 1992.

[8] L. Baum, T. Petrie, G. Soules, and N. Weiss. A maximization technique occurring in the statistical analysis of probabilistic functions of Markov chains. *Ann. Math. Statistics*, 41(1):164–171, 1970.

[9] L. E. Baum and T. Petrie. Statistical inference for probabilistic functions of finite state Markov chains. *Ann. Math. Statistics*, 37:1554–1563, 1966.

[10] F. Beckman. The solution of linear equations by the conjugate gradient method. In A. Ralston, H. Wilf, and K. Enslein, editors, *Mathematical Methods for Digital Computers*. Wiley, 1960.

[11] M. G. Bello. A combined Markov random field and wave-packet transform-based approach for image segmentation. *IEEE Trans. on Image Processing*, 3(6):834–846, November 1994.

[12] A. Benveniste, R. Nikoukhah, and A. Willsky. Multiscale system theory. In *Proceedings of the 29^{th} Conference on Decision and Control*, volume 4, pages 2484–2489, Honolulu, Hawaii, December 5-7 1990.

[13] J. Besag. Nearest-neighbour systems and the auto-logistic model for binary data. *Journal of the Royal Statistical Society B*, 34(1):75–83, 1972.

[14] J. Besag. Spatial interaction and the statistical analysis of lattice systems. *Journal of the Royal Statistical Society B*, 36(2):192–236, 1974.

[15] J. Besag. Efficiency of pseudolikelihood estimation for simple Gaussian fields. *Biometrica*, 64(3):616–618, 1977.

[16] J. Besag. On the statistical analysis of dirty pictures. *Journal of the Royal Statistical Society B*, 48(3):259–302, 1986.

[17] J. Besag. Towards Bayesian image analysis. *Journal of Applied Statistics*, 16(3):395–407, 1989.

[18] J. E. Besag and P. A. P. Moran. On the estimation and testing of spatial interaction in Gaussian lattice processes. *Biometrika*, 62(3):555–562, 1975.

[19] A. Blake. Comparison of the efficiency of deterministic and stochastic algorithms for visual reconstruction. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 11(1):2–30, January 1989.

[20] A. Blake and A. Zisserman. *Visual Reconstruction*. MIT Press, Cambridge, Massachusetts, 1987.

[21] C. A. Bouman and B. Liu. A multiple resolution approach to regularization. In *Proc. of SPIE Conf. on Visual Communications and Image Processing*, pages 512–520, Cambridge, MA, Nov. 9-11 1988.

[22] C. A. Bouman and B. Liu. Segmentation of textured images using a multiple resolution approach. In *Proc. of IEEE Int'l Conf. on Acoust., Speech and Sig. Proc.*, pages 1124–1127, New York, NY, April 11-14 1988.

[23] C. A. Bouman and B. Liu. Multiple resolution segmentation of textured images. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 13:99–113, Feb. 1991.

[24] C. A. Bouman and K. Sauer. An edge-preserving method for image reconstruction from integral projections. In *Proc. of the Conference on Information Sciences and Systems*, pages 382–387, Baltimore, MD, March 20-22 1991.

[25] C. A. Bouman and K. Sauer. A generalized Gaussian image model for edge-preserving map estimation. *IEEE Trans. on Image Processing*, 2:296–310, July 1993.

[26] C. A. Bouman and K. Sauer. Maximum likelihood scale estimation for a class of Markov random fields. In *Proc. of IEEE Int'l Conf. on Acoust., Speech and Sig. Proc.*, volume 5, pages 537–540, Adelaide, South Australia, April 19-22 1994.

[27] C. A. Bouman and K. Sauer. A unified approach to statistical tomography using coordinate descent optimization. *IEEE Trans. on Image Processing*, to appear in March 1996.

[28] C. A. Bouman and M. Shapiro. Multispectral image segmentation using a multiscale image model. In *Proc. of IEEE Int'l Conf. on Acoust., Speech and Sig. Proc.*, pages III–565 – III–568, San Francisco, California, March 23-26 1992.

[29] C. A. Bouman and M. Shapiro. A multiscale random field model for Bayesian image segmentation. *IEEE Trans. on Image Processing*, 3:162–177, March 1994.

[30] A. Brandt. *Multigrid Techniques: 1984 Guide with Applications to Fluid Dynamics*, volume Nr. 85. GMD-Studien, 1984. ISBN:3-88457-081-1.

[31] W. Briggs. *A Multigrid Tutorial*. Society for Industrial and Applied Mathematics, Philadelphia, 1987.

[32] P. Charbonnier, L. Blanc-Feraud, G. Aubert, and M. Barlaud. Two deterministic half-quadratic requlariza tion algorithms for computed imaging. In *Proc. of IEEE Int'l Conf. on Image Proc.*, volume 2, pages 168–176, Austin, TX, November, 13-16 1994.

[33] R. Chellappa and S. Chatterjee. Classification of textures using Gaussian Markov random fields. *IEEE Trans. on Acoust. Speech and Signal Proc.*, ASSP-33(4):959–963, August 1985.

[34] K. Chou, S. Golden, and A. Willsky. Modeling and estimation of multiscale stochastic processes. In *Proc. of IEEE Int'l Conf. on Acoust., Speech and Sig. Proc.*, pages 1709–1712, Toronto, Canada, May 14-17 1991.

[35] B. Claus and G. Chartier. Multiscale signal processing: Isotropic random fields on homogeneous trees. *IEEE Trans. on Circ. and Sys.:Analog and Dig. Signal Proc.*, 41(8):506–517, August 1994.

[36] F. Cohen and D. Cooper. Simple parallel hierarchical and relaxation algorithms for segmenting noncausal Markovian random fields. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, PAMI-9(2):195–219, March 1987.

[37] F. S. Cohen and Z. Fan. Maximum likelihood unsupervised texture image segmentation. *CVGIP:Graphical Models and Image Proc.*, 54(3):239–251, May 1992.

[38] F. S. Cohen, Z. Fan, and M. A. Patel. Classification of rotated and scaled textured images using Gaussian Markov random field models. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 13(2):192–202, February 1991.

[39] M. L. Comer and E. J. Delp. Parameter estimation and segmentation of noisy or textured images using the EM algorithm and MPM estimation. In *Proc. of IEEE Int'l Conf. on Image Proc.*, pages 650–654, Austin, Texas, November 1994.

[40] M. L. Comer and E. J. Delp. Multiresolution image segmentation. In *Proc. of IEEE Int'l Conf. on Acoust., Speech and Sig. Proc.*, pages 2415–2418, Detroit, Michigan, May 1995.

[41] D. B. Cooper. Maximum likelihood estimation of Markov process blob boundaries in noisy images. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, PAMI-1:372–384, 1979.

[42] D. B. Cooper, H. Elliott, F. Cohen, L. Reiss, and P. Symosek. Stochastic boundary estimation and object recognition. *Comput. Vision Graphics and Image Process.*, 12:326–356, April 1980.

[43] R. Cristi. Markov and recursive least squares methods for the estimation of data with discontinuities. *IEEE Trans. on Acoust. Speech and Signal Proc.*, 38(11):1972–1980, March 1990.

[44] G. R. Cross and A. K. Jain. Markov random field texture models. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, PAMI-5(1):25–39, January 1983.

[45] A. Dempster, N. Laird, and D. Rubin. Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society B*, 39(1):1–38, 1977.

[46] H. Derin and W. Cole. Segmentation of textured images using Gibbs random fields. *Comput. Vision Graphics and Image Process.*, 35:72–98, 1986.

[47] H. Derin, H. Elliot, R. Cristi, and D. Geman. Bayes smoothing algorithms for segmentation of binary images modeled by Markov random fields. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, PAMI-6(6):707–719, November 1984.

[48] H. Derin and H. Elliott. Modeling and segmentation of noisy and textured images using Gibbs random fields. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, PAMI-9(1):39–55, January 1987.

[49] H. Derin and P. A. Kelly. Discrete-index Markov-type random processes. *Proc. of the IEEE*, 77(10):1485–1510, October 1989.

[50] H. Derin and C. Won. A parallel image segmentation algorithm using relaxation with varying neighborhoods and its mapping to array processors. *Comput. Vision Graphics and Image Process.*, 40:54–78, October 1987.

[51] R. W. Dijkerman and R. R. Mazumdar. Wavelet representations of stochastic processes and multiresolution stochastic models. *IEEE Trans. on Signal Processing*, 42(7):1640–1652, July 1994.

[52] P. C. Doerschuk. Bayesian signal reconstruction, Markov random fields, and X-ray crystallography. *J. Opt. Soc. Am. A*, 8(8):1207–1221, 1991.

[53] R. Dubes and A. Jain. Random field models in image analysis. *Journal of Applied Statistics*, 16(2):131–164, 1989.

[54] R. Dubes, A. Jain, S. Nadabar, and C. Chen. MRF model-based algorithms for image segmentation. In *Proc. of the 10^{th} Internat. Conf. on Pattern Recognition*, pages 808–814, Atlantic City, NJ, June 1990.

[55] I. M. Elfadel and R. W. Picard. Gibbs random fields, cooccurrences and texture modeling. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 16(1):24–37, January 1994.

[56] H. Elliott, D. B. Cooper, F. S. Cohen, and P. F. Symosek. Implementation, interpretation, and analysis of a suboptimal boundary finding algorithm. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, PAMI-4(2):167–182, March 1982.

[57] W. Enkelmann. Investigations of multigrid algorithms. *Comput. Vision Graphics and Image Process.*, 43:150–177, 1988.

[58] N. P. Galatsanos and A. K. Katsaggelos. Methods for choosing the regularization parameter and estimating the noise variance in image restoration and their relation. *IEEE Trans. on Image Processing*, 1(3):322–336, July 1992.

[59] S. B. Gelfand and S. K. Mitter. Recursive stochastic algorithms for global optimization in R^d. *SIAM Journal on Control and Optimization*, 29(5):999–1018, September 1991.

[60] S. B. Gelfand and S. K. Mitter. Metropolis-type annealing algorithms for global optimization in R^d. *SIAM Journal on Control and Optimization*, 31(1):111–131, January 1993.

[61] S. B. Gelfand and S. K. Mitter. On sampling methods and annealing algorithms. In R. Chellappa and A. Jain, editors, *Markov Random Fields: Theory and Applications*, pages 499–515. Academic Press, Inc., Boston, 1993.

[62] D. Geman. Bayesian image analysis by adaptive annealing. In *Digest 1985 Int. Geoscience and Remote Sensing Symp.*, Amberst, MA, October 7-9 1985.

[63] D. Geman, S. Geman, C. Graffigne, and P. Dong. Boundary detection by constrained optimization. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 12(7):609–628, July 1990.

[64] D. Geman and G. Reynolds. Constrained restoration and the recovery of discontinuities. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 14(3):367–383, 1992.

[65] D. Geman, G. Reynolds, and C. Yang. Stochastic algorithms for restricted image spaces and experiments in deblurring. In R. Chellappa and A. Jain, editors, *Markov Random Fields: Theory and Applications*, pages 39–68. Academic Press, Inc., Boston, 1993.

[66] S. Geman and D. Geman. Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, PAMI-6:721–741, Nov. 1984.

[67] S. Geman and C. Graffigne. Markov random field image models and their applications to computer vision. In *Proc. of the Intl Congress of Mathematicians*, pages 1496–1517, Berkeley, California, 1986.

[68] S. Geman and D. McClure. Bayesian images analysis: An application to single photon emission tomography. In *Proc. Statist. Comput. sect. Amer. Stat. Assoc.*, pages 12–18, Washington, DC, 1985.

[69] S. Geman and D. McClure. Statistical methods for tomographic image reconstruction. *Bull. Int. Stat. Inst.*, LII-4:5–21, 1987.

[70] B. Gidas. A renormalization group approach to image processing problems. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 11(2):164–180, February 89.

[71] J. Goutsias. Unilateral approximation of Gibbs random field images. *CVGIP:Graphical Models and Image Proc.*, 53(3):240–257, May 1991.

[72] A. Gray, J. Kay, and D. Titterington. An empirical study of the simulation of various models used for images. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 16(5):507–513, May 1994.

[73] P. J. Green. Bayesian reconstruction from emission tomography data using a modified EM algorithm. *IEEE Trans. on Medical Imaging*, 9(1):84–93, March 1990.

[74] P. J. Green and X. liang Han. Metropolis methods, Gaussian proposals and antithetic variables. In P. Barone, A. Frigessi, and M. Piccioni, editors, *Stochastic Models, Statistical methods, and Algorithms in Image Analysis*, pages 142–164. Springer-Verlag, Berlin, 1992.

[75] M. I. Gurelli and L. Onural. On a parameter estimation method for Gibbs-Markov random fields. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 16(4):424–430, April 1994.

[76] W. Hackbusch. *Multi-Grid Methods and Applications*. Sprinter-Verlag, Berlin, 1980.

[77] K. Hanson and G. Wecksung. Bayesian approach to limited-angle reconstruction computed tomography. *J. Opt. Soc. Am.*, 73(11):1501–1509, November 1983.

[78] W. K. Hastings. Monte Carlo sampling methods using Markov chains and their applications. *Biometrika*, 57(1):97–109, 1970.

[79] T. Hebert and R. Leahy. A generalized EM algorithm for 3-D Bayesian reconstruction from Poisson data using Gibbs priors. *IEEE Trans. on Medical Imaging*, 8(2):194–202, June 1989.

[80] T. J. Hebert and K. Lu. Expectation-maximization algorithms, null spaces, and MAP image restoration. *IEEE Trans. on Image Processing*, 4(8):1084–1095, August 1995.

[81] F. Heitz and P. Bouthemy. Multimodal estimation of discontinuous optical flow using Markov random fields. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 15(12):1217–1232, December 1993.

[82] P. Huber. *Robust Statistics*. John Wiley & Sons, New York, 1981.

[83] B. Hunt. The application of constrained least squares estimation to image restoration by digital computer. *IEEE Trans. on Comput.*, c-22(9):805–812, September 1973.

[84] B. Hunt. Bayesian methods in nonlinear digital image restoration. *IEEE Trans. on Comput.*, c-26(3):219–229, 1977.

[85] J. Hutchinson, C. Koch, J. Luo, and C. Mead. Computing motion using analog and binary resistive networks. *Computer*, 21:53–63, March 1988.

[86] A. Jain. Advances in mathematical models for image processing. *Proc. of the IEEE*, 69:502–528, May 1981.

[87] B. D. Jeffs and M. Gunsay. Restoration of blurred star field images by maximally sparse optimization. *IEEE Trans. on Image Processing*, 2(2):202–211, April 1993.

[88] F.-C. Jeng and J. W. Woods. Compound Gauss-Markov random fields for image estimation. *IEEE Trans. on Signal Processing*, 39(3):683–697, March 1991.

[89] B. Jeon and D. Landgrebe. Classification with spatio-temporal interpixel class dependency contexts. *IEEE Trans. on Geoscience and Remote Sensing*, 30(4), JULY 1992.

[90] S. R. Kadaba, S. B. Gelfand, and R. L. Kashyap. Bayesian decision feedback for segmentation of binary images. In *Proc. of IEEE Int'l Conf. on Acoust., Speech and Sig. Proc.*, pages 2543–2546, Detroit, Michigan, May 8-12 1995.

[91] R. Kashyap and R. Chellappa. Estimation and choice of neighbors in spatial-interaction models of images. *IEEE Trans. on Information Theory*, IT-29(1):60–72, January 1983.

[92] R. Kashyap, R. Chellappa, and A. Khotanzad. Texture classification using features derived from random field models. *Pattern Recogn. Let.*, 1(1):43–50, October 1982.

[93] R. L. Kashyap and K.-B. Eom. Robust image modeling techniques with an image restoration application. *IEEE Trans. on Acoust. Speech and Signal Proc.*, 36(8):1313–1325, August 1988.

[94] Z. Kato, M. Berthod, and J. Zerubia. Parallel image classification using multiscale Markov random fields. In *Proc. of IEEE Int'l Conf. on Acoust., Speech and Sig. Proc.*, volume 5, pages 137–140, Minneapolis, MN, April 27-30 1993.

[95] A. Katsaggelos. Image identification and restoration based on the expectation-maximization algorithm. *Optical Engineering*, 29(5):436–445, May 1990.

[96] P. Kelly, H. Derin, and K. Hartt. Adaptive segmentation of speckled images using a hierarchical random field model. *IEEE Trans. on Acoust. Speech and Signal Proc.*, 36(10):1628–1641, October 1988.

[97] R. Kindermann and J. Snell. *Markov Random Fields and their Applications*. American Mathematical Society, Providence, 1980.

[98] J. Konrad and E. Dubois. Bayesian estimation of motion vector fields. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 14(9):910–927, September 1992.

[99] R. Lagendijk, A. Tekalp, and J. Biemond. Maximum likelihood image and blur identification: A unifying approach. *Optical Engineering*, 29(5):422–435, May 1990.

[100] R. L. Lagendijk, J. Biemond, and D. E. Boekee. Identification and restoration of noisy blurred images using the expectation-maximization algorithm. *IEEE Trans. on Acoust. Speech and Signal Proc.*, 38(7):1180–1191, July 1990.

[101] S. Lakshmanan and H. Derin. Simultaneous parameter estimation and segmentation of Gibbs random fields using simulated annealing. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 11(8):799–813, August 1989.

[102] S. Lakshmanan and H. Derin. Gaussian Markov random fields at multiple resolutions. In R. Chellappa and A. Jain, editors, *Markov Random Fields: Theory and Applications*, pages 131–157. Academic Press, Inc., Boston, 1993.

[103] S. Lakshmanan and H. Derin. Valid parameter space for 2-d Gaussian Markov random fields. *IEEE Trans. on Information Theory*, 39(2):703–709, March 1993.

[104] K. Lange. Convergence of EM image reconstruction algorithms with Gibbs smoothing. *IEEE Trans. on Medical Imaging*, 9(4):439–446, December 1990.

[105] K. Lange. An overview of Bayesian methods in image reconstruction. In *Proc. of the SPIE Conference on Digital Image Synthesis and Inverse Optics*, volume SPIE-1351, pages 270–287, San Diego, CA, 1990.

[106] S. Z. Li. *Markov Random Field Modeling in Computer Vision.* to be published, 1996.

[107] J. Liu and Y.-H. Yang. Multiresolution color image segmentation. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 16(7):689–700, July 1994.

[108] M. R. Luettgen, W. C. Karl, and A. S. Willsky. Efficient multiscale regularization with applications to the computation of optical flow. *IEEE Trans. on Image Processing*, 3(1), January 1994.

[109] M. R. Luettgen, W. C. Karl, A. S. Willsky, and R. R. Tenney. Multiscale representations of Markov random fields. *IEEE Trans. on Signal Processing*, 41(12), December 1993. special issue on Wavelets and Signal Processing.

[110] M. R. Luettgen and A. S. Willsky. Likelihood calculation for a class of multiscale stochastic models, with application to texture discrimination. *IEEE Trans. on Image Processing*, 4(2):194–207, February 1995.

[111] B. Manjunath, T. Simchony, and R. Chellappa. Stochastic and deterministic networks for texture segmentation. *IEEE Trans. on Acoust. Speech and Signal Proc.*, 38(6):1039–1049, June 1990.

[112] J. Mao and A. K. Jain. Texture classification and segmentation using multiresolution simultaneous autoregressive models. *Pattern Recognition*, 25(2):173–188, 1992.

[113] J. Marroquin, S. Mitter, and T. Poggio. Probabilistic solution of ill-posed problems in computational vision. *Journal of the American Statistical Association*, 82:76–89, March 1987.

[114] S. McCormick, editor. *Multigrid Methods*. Society for Industrial and Applied Mathematics, Philadelphia, 1987.

[115] N. Metropolis, A. Rosenbluth, M. Rosenbluth, A. Teller, and E. Teller. Equations of state calculations by fast computing machines. *J. Chem. Phys.*, 21:1087–1091, 1953.

[116] J. W. Modestino and J. Zhang. A Markov random field model-based approach to image interpretation. In R. Chellappa and A. Jain, editors, *Markov Random Fields: Theory and Applications*, pages 369–408. Academic Press, Inc., Boston, 1993.

[117] H. H. Nguyen and P. Cohen. Gibbs random fields, fuzzy clustering, and the unsupervised segmentation of textured images. *CVGIP:Graphical Models and Image Proc.*, 55(1):1–19, January 1993.

[118] Y. Ogata. A Monte Carlo method for an objective Bayesian procedure. *Ann. Inst. of Statist. Math.*, 42(3):403–433, 1990.

[119] L. Onsager. Crystal statistics I. a two-dimensional model. *Physical Review*, 65:117–149, 1944.

[120] L. Onural. Generating connected textured fractal patterns using Markov random fields. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 13(8):819–825, August 1991.

[121] L. Onural, M. B. Alp, and M. I. Gurelli. Gibbs random field model based weight selection for the 2-d adaptive weighted median filter. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 16(8):831–837, August 1994.

[122] T. Pappas. An adaptive clustering algorithm for image segmentation. *IEEE Trans. on Signal Processing*, 4:901–914, April 1992.

[123] R. E. Peierls. On Ising's model of ferromagnetism. *Proc. Camb. Phil. Soc.*, 32:477–481, 1936.

[124] R. E. Peierls. Statistical theory of adsorption with interaction between the adsorbed atoms. *Proc. Camb. Phil. Soc.*, 32:471–476, 1936.

[125] P. Perez and F. Heitz. Multiscale Markov random fields and constrained relaxation in low level image analysis. In *Proc. of IEEE Int'l Conf. on Acoust., Speech and Sig. Proc.*, volume 3, pages 61–64, San Francisco, CA, March 23-26 1992.

[126] P. H. Peskun. Optimum Monte-Carlo sampling using Markov chains. *Biometrika*, 60(3):607–612, 1973.

[127] D. Pickard. Asymptotic inference for an ising lattice III. non-zero field and ferromagnetic states. *J. Appl. Prob.*, 16:12–24, 1979.

[128] D. Pickard. Inference for discrete Markov fields: The simplest nontrivial case. *Journal of the American Statistical Association*, 82:90–96, March 1987.

[129] D. N. Politis. Markov chains in many dimensions. *Adv. Appl. Prob.*, 26:756–774, 1994.

[130] W. H. Pun and B. D. Jeffs. Data directed MAP image restoration using an adaptive generalized Gaussian-Markov random field model. In *(submitted to)Proc. of IEEE Int'l Conf. on Acoust., Speech and Sig. Proc.*, Atlanta, GA, May 7-10 1996.

[131] L. R. Rabiner. A tutorial on hidden Markov models and selected applications in speech recognition. *Proc. of the IEEE*, 77(2):257–286, February 1989.

[132] L. R. Rabiner and B. H. Juang. An introduction to hidden Markov models. *IEEE Signal Proc. Magazine*, 3(1):4–16, January 1986.

[133] E. Redner and H. Walker. Mixture densities, maximum likelihood and the EM algorithm. *SIAM Review*, 26(2), April 1984.

[134] S. Reeves and R. Mersereau. Optimal estimation of the regularization parameter and stabilizing fu nctional for regularized image restoration. *Optical Engineering*, 29(5):446–454, May 1990.

[135] S. J. Reeves and R. M. Mersereau. Blur identification by the method of generalized cross-validation. *IEEE Trans. on Image Processing*, 1(3):301–321, July 1992.

[136] W. J. J. Rey. *Introduction to Robust and Quasi-Robust Statistical Methods.* Springer-Verlag, Berlin, 1980.

[137] E. Rignot and R. Chellappa. Segmentation of polarimetric synthetic aperature radar data. *IEEE Trans. on Image Processing*, 1(3):281=300, July 1992.

[138] B. D. Ripley. *Stochastic Simulation.* John Wiley & Sons, New York, 1987.

[139] J. Rissanen. A universal prior for integers and estimation by minimum description length. *Annals of Statistics*, 11(2):417–431, 1983.

[140] B. Rouchouze, P. Mathieu, T. Gaidon, and M. Barlaud. Motion estimation based on Markov random fields. In *Proc. of IEEE Int'l Conf. on Image Proc.*, volume 3, pages 270–274, Austin, TX, November, 13-16 1994.

[141] S. S. Saquib, C. A. Bouman, and K. Sauer. ML parameter estimation for markov random fields, with applications to bayesian tomography. Technical Report TR-ECE 95-24, School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907, 1995.

[142] S. S. Saquib, C. A. Bouman, and K. Sauer. Efficient ML estimation of the shape parameter for generalized Gaussian MRF. In *(submitted to)Proc. of IEEE Int'l Conf. on Acoust., Speech and Sig. Proc.*, Atlanta, GA, May 7-10 1996.

[143] K. Sauer and C. Bouman. Bayesian estimation of transmission tomograms using segmentation based optimization. *IEEE Trans. on Nuclear Science*, 39:1144–1152, 1992.

[144] K. Sauer and C. A. Bouman. A local update strategy for iterative reconstruction from projections. *IEEE Trans. on Signal Processing*, 41(2), February 1993.

[145] R. Schultz, R. Stevenson, and A. Lumsdaine. Maximum likelihood parameter estimation for non-Gaussian prior signal models. In *Proc. of IEEE Int'l Conf. on Image Proc.*, volume 2, pages 700–704, Austin, TX, November 1994.

[146] R. R. Schultz and R. L. Stevenson. A Bayesian approach to image expansion for improved definition. *IEEE Trans. on Image Processing*, 3(3):233–242, May 1994.

[147] R. R. Schultz and R. L. Stevenson. Stochastic modeling and estimation of multispectral image data. *IEEE Trans. on Image Processing*, 4(8):1109–1119, August 1995.

[148] J. Silverman and D. Cooper. Bayesian clustering for unsupervised estimation of surface and texture models. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 10(4):482–495, July 1988.

[149] T. Simchony, R. Chellappa, and Z. Lichtenstein. Relaxation algorithms for MAP estimation of gray-level images with multiplicative noise. *IEEE Trans. on Information Theory*, 36(3):608–613, May 1990.

[150] S. Sista, C. A. Bouman, and J. Allebach. Fast image search using a multiscale stochastic model. In *Proc. of IEEE Int'l Conf. on Image Proc.*, Washington, DC, October 22-25 1995.

[151] R. Stevenson and E. Delp. Fitting curves with discontinuities. *Proc. of the first international workshop on robust computer vision*, pages 127–136, October 1-3 1990.

[152] H. Tan, S. Gelfand, and E. Delp. A comparative cost function approach to edge detection. *IEEE Trans. on Systems Man and Cybernetics*, 19(6):1337–1349, December 1989.

[153] T. Taxt, P. Flynn, and A. Jain. Segmentation of document images. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 11(12):1322–1329, December 1989.

[154] D. Terzopoulos. Image analysis using multigrid relaxation methods. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, PAMI-8(2):129–139, March 1986.

[155] D. Terzopoulos. The computation of visible-surface representations. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 10(4):417–438, July 1988.

[156] C. Therrien. An estimation-theoretic approach to terrain image segmentation. *Comput. Vision Graphics and Image Process.*, 22:313–326, 1983.

[157] C. Therrien, T. Quatieri, and D. Dudgeon. Statistical model-based algorithm for image analysis. *Proc. of the IEEE*, 74(4):532–551, April 1986.

[158] C. W. Therrien, editor. *Decision Estimation and Classification: An Introduction to Pattern Recognition and Related Topics*. John Wiley & Sons, New York, 1989.

[159] A. Thompson, J. Kay, and D. Titterington. A cautionary note about the cross validatory choice. *J. statist. Comput. Simul.*, 33:199–216, 1989.

[160] A. M. Thompson, J. C. Brown, J. W. Kay, and D. M. Titterington. A study of methods of choosing the smoothing parameter in image restoration. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 13(4):326–339, 1991.

[161] A. Tikhonov and V. Arsenin. *Solutions of Ill-Posed Problems*. Winston and Sons, New York, 1977.

[162] D. Tretter, C. A. Bouman, K. Khawaja, and A. Maciejewski. A multiscale stochastic image model for automated inspection. *IEEE Trans. on Image Processing*, to appear December 1995.

[163] C. Won and H. Derin. Segmentation of noisy textured images using simulated annealing. In *Proc. of IEEE Int'l Conf. on Acoust., Speech and Sig. Proc.*, pages 14.4.1–14.4.4, Dallas, TX, 1987.

[164] C. S. Won and H. Derin. Unsupervised segmentation of noisy and textured images using Markov random fields. *CVGIP:Graphical Models and Image Proc.*, 54(4):308–328, July 1992.

[165] J. Woods, S. Dravida, and R. Mediavilla. Image estimation using double stochastic Gaussian random field models. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, PAMI-9(2):245–253, March 1987.

[166] C. Wu. On the convergence properties of the EM algorithm. *Annals of Statistics*, 11(1):95–103, 1983.

[167] C.-h. Wu and P. C. Doerschuk. Cluster expansions for the deterministic computation of Bayesian estimators based on Markov random fields. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 17(3):275–293, March 1995.

[168] C.-h. Wu and P. C. Doerschuk. Texture-based segmentation using Markov random field models and approximate Bayesian estimators based on trees. *J. Math. Imaging and Vision*, 5(4), December 1995.

[169] C.-h. Wu and P. C. Doerschuk. Tree approximations to Markov random fields. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 17(4):391–402, April 1995.

[170] Z. Wu and R. Leahy. An approximate method of evaluating the joint likelihood for first-order GMRFs. *IEEE Trans. on Image Processing*, 2(4):520–523, October 1993.

[171] J. Zerubia and R. Chellappa. Mean field annealing using compound Gauss-Markov random fields for edge detection and image estimation. *IEEE Trans. on Neural Networks*, 4(4):703–709, July 1993.

[172] J. Zhang. The mean field theory in EM procedures for Markov random fields. *IEEE Trans. on Signal Processing*, 40(10):2570–2583, October 1992.

[173] J. Zhang. The mean field theory in EM procedures for blind Markov random field image restoration. *IEEE Trans. on Image Processing*, 2(1):27–40, January 1993.

[174] J. Zhang and G. G. Hanauer. The application of mean field theory to image motion estimation. *IEEE Trans. on Image Processing*, 4(1):19–33, January 1995.

[175] J. Zhang and J. W. Modestino. A model-fitting approach to cluster validation with application to stochastic model-based image segmentation. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 12(10):1009–1017, October 1990.

[176] J. Zhang, J. W. Modestino, and D. A. Langan. Maximum-ikelihood parameter estimation for unsupervised stochastic model-based image segmentation. *IEEE Trans. on Image Processing*, 3(4):404–420, July 1994.

[177] M. Zhang, R. Haralick, and J. Campbell. Multispectral image context classification using stochastic relaxation. *IEEE Trans. on Systems Man and Cybernetics*, vol. 20(1):128–140, February 1990.