EE 641 DIGITAL IMAGE PROCESSING II

Assignment #5 - Spring 1996

Monday March 12, 1996

Problem 1.

$\{X_n\}_{n=-\infty}^{\infty}$ is a 1^{st} order GMRF with zero mean and autocorrelation

$$E[X_n X_{n+k}] = r_k .$$

Compute $E[X_n | X_i \; i \neq n]$ and $VAR[X_n | X_i \; i \neq n]$ in terms of r_k.

Problem 2.

Let $\{X_n\}_{n=-\infty}^{\infty}$ be a 1^{st} order zero mean Gaussian AR Process parameterized by the prediction variance σ_c^2 and prediction filter h.

a) Compute the density functions $p(x_2, \ldots, x_N | x_1)$ and $p(x_1, \ldots, x_N)$.

b) Assume that σ_c^2 is known and compute

$$\hat{h} = \arg\max_h p_h(x_2, \ldots, x_N | x_1) \; .$$

Problem 3.

Let $\{X_n\}_{n=1}^{N}$ be a homogeneous discrete valued Markov chain with

$$X_n \in \{1, \ldots, M\}$$

transition matrix

$$P_{i,j} = P\{X_n = j | X_{n-1} = i\}$$

and initial distribution

$$P\{X_1 = i\} = 1/M \; .$$

Use the density function for $\{X_n\}_{n=1}^{N}$ and the fact that

$$P_{i,i} = 1 - \sum_{j \neq i} P_{i,j}$$

to compute the ML estimate of all components $P_{i,j}$ for $i \neq j$.

Problem 4.

Let X be a Ising model with each $X_s \in \{-1, 1\}$. Let N be zero mean white Gaussian noise with variance σ^2, and let Y be an observed image

$$Y = X + N \; .$$

a) Compute an expression for the MAP estimate of X given Y.

b) Give any algorithm for the approximate computation of the MAP estimate. Be specific.

Problem 5.

Let $\{X_n\}_{n=1}^N$ be a 1D discrete valued MRF with neighborhood system

$$\partial n = \{n-1, n+1\} \cap \{1, \ldots, N\}$$

and strictly positive distribution. Prove that $\{X_n\}_{n=1}^N$ is also a Markov Chain.