EE 641 DIGITAL IMAGE PROCESSING II
Assignment #3 - Spring 1996
Tuesday February 20, 1996

1) Let Y be a 1-D AR process with $h_n = \rho\delta_{n-1}$ and σ^2 prediction variance. Compute (σ^2_{NC}, g) the noncausal prediction variance and the noncausal prediction filter.

2) Let Y and X be random variables, and let Y_{MAP} and Y_{MMSE} be the MAP and MMSE estimates respectively of Y given X. Pick distributions for Y and X so that the MAP estimator is very "poor", but the MMSE estimator is "good".

3) Find a convex function $f(x_1, x_2)$ with a unique global minimum, so that coordinate decent does not converge to the global minimum. Why?

4) Consider the following functional

$$f(x) = (b - Ax)^t B(b - Ax)$$

where b and x are vectors, and A and B are matrices where B is symmetric and positive definite.
a) Compute the Gradient decent algorithm for minimizing $f(x)$.
b) Compute the Coordinate decent algorithm for minimizing $f(x)$.
c) Compute the Steepest decent algorithm for minimizing $f(x)$.

5) Let Y be a 1-D order p GMRF with

$$\begin{aligned} \hat{Y_n} &= E[Y_n | Y_i \; i < n] \\ &= \sum_{j=-p}^{p} g_j Y_{n-j} \\ E[(Y_n - \hat{Y_n})^2] &= \sigma^2_{nc} \; . \end{aligned}$$

Show that

$$p(y) = \frac{1}{(2\pi\sigma^2_{nc})^{(N/2)}} |B|^{1/2} \exp\left\{ \frac{1}{2\sigma^2_{nc}} y^t B y \right\}$$

where

$$B_{i,j} = \delta_{i-j} - g_{i-j} \; .$$