EE 641 DIGITAL IMAGE PROCESSING II
Assignment #1 - Spring 1996
January 25, 1996

1) Let $\{x_i\}_{i=1}^N$ be iid RV's with distribution

$$\begin{aligned} P(x_i = 1) &= \theta \\ P(x_i = 0) &= 1 - \theta \end{aligned}$$

Compute the ML estimate of θ.

2) Let X, N, and Y be Gaussian random vectors such that $X \sim N(0, R_x)$ and $N \sim N(0, R_n)$, and let θ be a deterministic vector.
a) Compute the ML estimate of θ when $Y = \theta + N$.
b) Compute the MSEE estimate of X when $Y = X + N$.

3) Let Y be a 1-D AR process with $h_n = \rho\delta_{n-1}$ and σ^2 prediction variance.
a) Analytically calculate $S_y(\omega)$ (the power of Y) and $R_y(n)$ (the autocorrelation function for Y).
a) Plot $S_y(\omega)$ and $R_y(n)$ for $\rho = 0.5$ and $\rho = 0.95$.

4) Let Y_n be samples of an AR process with order p and parameters (σ^2, h). Also make the assumption that $Y_n = 0$ for $n \leq 0$ as we did in class.
a) Use matlab to generate 100 samples of Y. Experiment with a variety of values for p and (σ^2, h). Plot your output for each experiment.
b) Use your sample values of Y generated in part a) to compute the ML estimates of the (σ^2, h), and compare them to the true values.

6) You are at dinner, and you overhear a student from the University of Illinios say that if X, Y, and Z are independent Gaussian randon variables that are pairwise independent, than X is independent of the vector (Y, Z). Either prove this statement or give a counter example.