0/15 Questions Answered

TIME REMAINING
2 hrs 59 mins

ECE641-F2020-Final

Q1

2 Points

Rules: I understand that this is an open book exam that shall be done within the allotted time of 180 minutes. I can use my notes, previous posted exams and exam solutions, and web resources. However, I will not communicate with any other person other than the official exam proctors during the exam, and I will not seek or accept help from any other persons other than the official proctors.

Upload a scan of your signature here:

Please select file(s) Select file(s)

Q2 Reversible Markov Chains

21 Points

Let $\{X_n\}_{n=0}^{\infty}$ be a homogeneous Markov chain with states $\{0, \cdots, M-1\}$, transition probabilities $P_{i,j}$, and state distribution $P\{X_n = i\} = \pi_i$ for all n.

For the purposes of this problem, we say that X_n is reversible if and only if for all $n > 0$,

$$P\{X_n = i, X_{n-1} = j\} = P\{X_n = j, X_{n-1} = i\}$$

Q2.1

7 Points

Prove the DBE equations are satisfied if and only if X_n is reversible.

Choose Files No file chosen

Please select file(s) Select file(s)

Q2.2

7 Points

Prove that if the DBE equations hold for some π_i and $P_{i,j}$, then the FBE equations must also hold.

Please select file(s) Select file(s)

Q2.3

7 Points

Assume that the DBE equations hold and that X_n is reversible, and define $P^\infty = \lim_{n\to\infty} P^n$.

Then is it always the case that P^∞ has the following form?

$$P^\infty = \begin{bmatrix} \pi \\ \vdots \\ \pi \end{bmatrix}$$

If so, then prove it is true.
Otherwise, give a counter example.

Enter your answer here

Please select file(s) Select file(s)

Q3 Birth Death Processes

28 Points

Let $\{X_n\}_{n=0}^\infty$ be a Markov chain with states $\{0, \cdots, M-1\}$, transition probabilities $P_{i,j}$, and initial distribution $P\{X_0 = i\} = \tau_i$. Furthermore, let

Choose Files No file chosen

$$P_{i,j} = \begin{cases} \lambda & \text{if } j = i+1 \text{ and } i \geq 0 \\ \mu & \text{if } j = i-1 \text{ and } i > 0 \\ 1-\lambda-\mu & \text{if } j = i \text{ and } i > 0 \\ 1-\lambda & \text{if } j = i \text{ and } i = 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\lambda > 0$, $\mu > 0$, and $\lambda + \mu < 1$.

Q3.1

7 Points

Prove that X_n is a homogeneous Markov chain.

Please select file(s) Select file(s)

Q3.2

7 Points

Prove that X_n is an irreducible Markov chain.

Please select file(s) Select file(s)

Q3.3

7 Points

Assume that $\lambda < \mu$, then prove that X_n is reversible and find its stationary distribution π_i.

Please select file(s) Select file(s)

Q3.4

7 Points

For which values of λ and μ is X_n **not** ergodic?
Justify your answer and provide an intuitive explanation.

Please select file(s) Select file(s)

Choose Files No file chosen

Q4 Markov Random Fields

21 Points

Consider a random field X_s with distribution

$$p(x) = \frac{1}{z} \exp \left\{ - \sum_{\{s,r\} \in P} b_{s,r} \rho(x_s - x_r) \right\}$$

where P is the set of neighboring pixel pairs on a finite rectangular lattice S and neighborhood system ∂s.

Q4.1

7 Points

Calculate the conditional probability $p(x_s | x_r \text{ for } r \neq s)$, and use it to show that X_s is a MRF.

Please select file(s) Select file(s)

Q4.2

7 Points

If $\rho(\Delta) = \frac{1}{2}|\Delta|^2$, then what type of prior distribution is this?

Also, what advantages and disadvantages does this choice have?

Please select file(s) Select file(s)

Q4.3

7 Points

If $\rho(\Delta) = |\Delta|$, then what type of prior distribution is this?

Also, what advantages and disadvantages does this choice have?

Please select file(s) Select file(s)

Choose Files No file chosen

Q5 EM Algorithm

28 Points

Let $\{X_n\}_{n=1}^N$ be i.i.d. random variables with $P\{X_n = i\} = \pi_i$ for $i \in \{0, \cdots, M-1\}$. Also, assume that Y_n are conditionally independent Gaussian random variables given X_n and that the conditional distribution of Y_n given X_n is distributed as $N(\mu_{X_n}, \sigma^2)$. Furthermore, let $\theta = (\pi_0, \mu_0, \cdots, \pi_{M-1}, \mu_{M-1})$ parameterize the distribution.

Q5.1

7 Points

What is the name of the distribution of Y_n?

Enter your answer here

Q5.2

7 Points

If you observe both $\{X_n, Y_n\}_{n=1}^N$, then what is the ML estimate of θ?

Please select file(s) Select file(s)

Q5.3

7 Points

If you observe only $\{Y_n\}_{n=1}^N$, then what is the EM update for computing the ML estimate of θ?

Please select file(s) Select file(s)

Q5.4

7 Points

Does the EM algorithm always converge to a global maximum of the likelihood for this problem?

If so, why? If not, why not?

Please select file(s) Select file(s)

Submit & View Submission