EE 641 Midterm Exam

October 12, Fall 2012

Name: ______________________________

Instructions

The following is an in-class closed-book exam.

- This exam contains 3 problems worth a total of 108 points.
- You may not use any notes, textbooks, or calculators.

Good luck.

Problem 1. (36pt)
Let $Y \in \mathbb{R}^M$ and $X \in \mathbb{R}^N$ be zero-mean jointly Gaussian random vectors. Then define the following notation for this problem. Let $p(y,x)$ and $p(y|x)$ be the joint and conditional density of Y given X. Let B be the joint positive-definite precision matrix (i.e., inverse covariance matrix) given by $B^{-1} = \mathbb{E}[ZZ^t]$ where $Z = \begin{bmatrix} Y \\ X \end{bmatrix}$. Furthermore, let C, D, and E be the matrix blocks that form B, so that

$$B = \begin{bmatrix} C & D \\ D^t & E \end{bmatrix} .$$

where $C \in \mathbb{R}^{M\times M}$, $D \in \mathbb{R}^{M\times N}$, and $E \in \mathbb{R}^{N\times N}$. Finally, define the matrix A so that $AX = \mathbb{E}[Y|X]$ and the matrix Λ so that $\Lambda^{-1} = \mathbb{E}[(Y - AX)(Y - AX)^t|X]$.
a) Write out an expression for $p(y,x)$ in terms of B.
b) Write out an expression for $p(y|x)$ in terms of A and Λ.
c) Derive an expression for Λ the in terms of C, D, and E.
d) Derive an expression for A the in terms of C, D, and E.

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Problem 2. (36pt)

Let X_n be a 1-D zero-mean Gaussian AR process with MMSE causal prediction filter given by $h_n = \rho\delta_{n-1}$ and causal prediction variance σ_C^2.

a) Calculate, $S_X(\omega)$, the power spectral density of the random process.

b) Calculate, $R_X(n)$, the time autocorrelation of the random process.

c) Calculate (σ_{NC}^2, g_n) the noncausal prediction variance and the noncausal prediction filter for the equivalent GMRF. Hint: Remember that

$$\sigma_{NC}^2(\delta_n - h_n) * (\delta_n - h_{-n}) = \sigma_C^2(\delta_n - g_n) \ .$$

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Problem 3. (36pt)

Consider the optimization problem

$$\hat{x} = \arg \min_{x \in \mathbb{R}^N} \left\{ ||y - Ax||_\Lambda^2 + x^t B x \right\}$$

where A is a nonsingular $N \times N$ matrix, B is a positive-definite $N \times N$ matrix, and Λ is a diagonal and positive-definite matrix..

a) Derive a closed form expression for the solution.

b) Calculate an expression for the gradient descent update using step size $\mu \geq 0$.

c) Calculate an expression for the coordinate descent update.

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