EE 641 Final Exam

Fall 2006

Name: ______________________________ Starting time: ________________________

Ending time: ________________________

**Instructions**

The following are important rules for this take home exam.

- Accurately fill in a start time and ending time for your exam.
- You are allowed 24 hours to complete the exam. Hand in the exam after that period **whether or not you have completed it.**
- You should not discuss these problems with any other person. In addition, you should not communicate with any other student in the class during the test period.
- Your are allowed to use all class notes and handouts (including notes from homeworks and labs), any material posted on the official course web page, and the course text book from EE600 or an equivalent graduate course.
- Your are not allowed to use any other supplementary information, including sources from the library, publications not handed out in class, or google searches from the web.
- You must hand in your completed exam by Monday December 18 at 9:00AM.
- If you can not physically hand in your exam, you may email me a scanned copy of the exam. Please make sure the scan is readable, and make sure you get absolute confirmation of my receipt of the exam.
- If you have any questions, call me at home 463-4378 (8:00AM to 8:00PM); or office 494-0340, or send email at bouman@purdue.edu.

Good luck.

**Problem 1.**(33pt)

a) Let $X_n$ for $n = 0$ to $N-1$ be i.i.d. scalar Gaussian random variables with unknown mean $\mu$ and variance $\sigma^2 = 1$. Derive the maximum likelihood estimate for $\mu$.

b) Let $X_n$ for $n = 0$ to $N-1$ be i.i.d. scalar Gaussian random variables with unknown mean $\mu$ and unknown variance $\sigma^2$. Derive the maximum likelihood estimate for $\mu$ and $\sigma^2$.

c) Let $X_n$ for $n = 0$ to $N-1$ be i.i.d. $M$ dimensional Gaussian random vectors with unknown mean $\mu$ and convariance $R$. Derive the maximum likelihood estimate for $\mu$ and $R$.

**Problem 2.**(33pt) Let $X_n$ be i.i.d. random variables with probability mass function given by

$$P\{X_n = m\} = \pi_m$$

where $\sum_{m=0}^{M-1} \pi_m = 1$. Further more, let $Y_n$ be conditionally i.i.d. given $X_n$ with conditional density given by

$$p(y_n|x_n) = \theta_{x_n} \exp\{-y_n \theta_{x_n}\}$$

for $0 \leq n < N$ where $\theta = [\theta_0, \theta_1, \cdots, \theta_{M-1}]$.

a) Calculate an expression for $p(y, x)$ where $y = (y_0, y_1, \cdots y_{N-1})$ and $x = (x_0, x_1, \cdots x_{N-1})$.

b) Calculate an expression for $p(y)$ where $y = (y_0, y_1, \cdots y_{N-1})$.

c) Calculate the EM update for computing the ML estimate of $\theta$ from observations of $y$.

**Problem 3.**(34pt) Let $X_s$ for $s = (s_1, s_2)$ for $0 \leq s_1 < N$ and $0 \leq s_2 < N$ be an $N \times N$ MRF taking values $X_s \in \{0, \cdots, M-1\}$, and having Gibbs distribution

$$p(x) = \frac{1}{z} \exp \left\{ -\beta \sum_{\{s,r\} \in \mathcal{C}} \delta(x_s \neq x_r) \right\}$$

where $\mathcal{C}$ is the set of all pairs of 4pt nearest neighbors using a toriodal boundary. Furthermore, let $Y_s$ be i.i.d. Gaussian random variables with conditional mean $\mu_{X_s}$ and conditional variance of 1.

a) Calculate an expression for

$$-\log p(y|x) - \log p(x)$$

b) Caculate the expressions and give the algorithm for computing the approximate MAP estimate using the ICM algorithm.

c) Do the ICM updates converge? Why or why not? If so, what do they converge to?