EE 641 DIGITAL IMAGE PROCESSING II
Assignment #2 - Fall 2000

1. Run class demos for
   a) Optimization methods
   b) Markov Chains and processes

2. Let $f : I\!R^N \rightarrow I\!R$ and $g : I\!R^N \rightarrow I\!R$ be convex functions, and let A be an $N \times M$ matrix. Prove that
   a) $h(x) = f(Ax)$ for $x \in I\!R^M$ is a convex function.
   b) $h(x) = f(x) + g(x)$ is a convex function.
   c) $f(x)$ is continuous.

3. Let $f : I\!R^N \rightarrow I\!R$ be a strictly convex continuously differentiable function. Prove that if $\bigtriangledown f(x^*) = 0$ if and only if x^* is the unique global minimum of $f(\cdot)$.

4. Let X_i be i.i.d. random variables with $P\{X_i = k\} = \pi_k$ for $k = 0, \cdots, M-1$. Also, assume that Y_i are conditionally independent given X with $p(y_i|x_i) \sim N(\mu_{x_i}, \gamma_{x_i})$. Derive a EM algorithm algorithm for estimating the parameters $\theta = (\pi, \mu, \gamma)$.

5. Let $\{X_i\}_{i=0}^N$ be a Markov Chain with $X_i \in \{0, \cdots, M-1\}$ with transition probabilities given by
   $$P\{X_i = k | X_{i-1} = l\} = P_{l,k}$$
   where $0 \leq k, l < M$, and initial condition $P\{X_0 = k\} = \pi_k$. Compute an expression for the ML estimates of $\theta = (\pi, P)$.

6. Let Y be a 1-D AR process with $h_n = \rho\delta_{n-1}$ and σ^2 prediction variance. Compute (σ^2_{NC}, g) the noncausal prediction variance and the noncausal prediction filter.