EE 641 DIGITAL IMAGE PROCESSING II
Assignment #1 - Fall 2000

1) Let $\{x_i\}_{i=1}^N$ be iid RV's with distribution

$$\begin{aligned} P(x_i = 1) &= \theta \\ P(x_i = 0) &= 1 - \theta \end{aligned}$$

Compute the ML estimate of θ.

2) Let X, N, and Y be Gaussian random vectors such that $X \sim N(0, R_x)$ and $N \sim N(0, R_n)$, and let θ be a deterministic vector.
a) Compute the ML estimate of θ when $Y = \theta + N$.
b) Compute the MSEE estimate of X when $Y = X + N$.

3) Let $\{Y_n\}_{n=1}^N$ be a 1-D Gaussian random process such that

$$X_n = Y_n - \sum_{i=n-p}^{n-1} h_{n-i} Y_i$$

results in X_n being i.i.d. $N(0, \sigma^2)$ random variables for $n = 1 \cdots N$, and Y_n is assumed to be 0 for $n \leq 0$. Compute the ML estimates of the prediction filter h_n and the prediction variance σ^2.

4) Let Y_n be samples of an AR process with order p and parameters (σ^2, h). Also make the assumption that $Y_n = 0$ for $n \leq 0$ as we did in class.
a) Use matlab to generate 100 samples of Y. Experiment with a variety of values for p and (σ^2, h). Plot your output for each experiment.
b) Use your sample values of Y generated in part a) to compute the ML estimates of the (σ^2, h), and compare them to the true values.

5) Let Y be a 1-D AR process with $h_n = \rho\delta_{n-1}$ and σ^2 prediction variance.
a) Analytically calculate $S_y(\omega)$ (the power of Y) and $R_y(n)$ (the autocorrelation function for Y).
a) Plot $S_y(\omega)$ and $R_y(n)$ for $\rho = 0.5$ and $\rho = 0.95$.

6) Let $\{Y_n\}_{n=0}^{N-1}$ be a 1-D order p GMRF with

$$\begin{aligned} \hat{Y_n} &= E[Y_n | Y_i \ i \neq n] \\ &= \sum_{j=-p}^{p} g_j Y_{n-j} \\ E[(Y_n - \hat{Y_n})^2] &= \sigma_{nc}^2 \end{aligned}$$

where Y_n is assumed 0 for $n < 0$ or $n \geq N$. Show that

$$p(y) = \frac{1}{(2\pi\sigma_{nc}^2)^{(N/2)}} |B|^{1/2} \exp\left\{ \frac{1}{2\sigma_{nc}^2} y^t B y \right\}$$

where

$$B_{i,j} = \delta_{i-j} - g_{i-j} \ .$$