EE 438 Digital Signal Processing with Applications
Homework #9 due 4/26/99

1. Consider the following 2-D difference equation for $|a| \leq 1$.

$$y(n,m) = x(n,m) + ay(n-1,m) + ay(n,m-1) - a^2 y(n-1,m-1)$$

   a) Calculate the 2-D transfer function of the system $H(e^{j\mu}, e^{j\nu})$.
   b) Show that $H(e^{j\mu}, e^{j\nu})$ is a separable system, or show that it is not.
   c) Sketch the 2-D magnitude $\left|H(e^{j\mu}, e^{j\nu})\right|$ for *a* = 0.9.
   d) Calculate the stable 2-D impulse response *h*(*n*,*m*).

2. A CCD imaging system is composed of a focusing lens followed by a CCD detector. The point spread function of the lens is $\text{jinc}(2x, 2y)$ with a magnifications of $M = 1$, and the CCD has a cell aperture of $a = 0.5$ and a cell spacing of $T_x = T_y = 0.5$.

   a. Compute the effective spatial frequency response $H(u,v) = \dfrac{\tilde{G}(u,v)}{F(u,v)}$ for the combined effect of the lens and CCD aperature.
   b. Assuming the input image is $f(x,y) = \delta(x,y)$, compute **and sketch** the sampled image spectrum $G(e^{j\mu}, e^{j\gamma})$.
   c. Is there any aliasing? Why or why not.

3. A monochrome image is obtained by measuring the normalized energy incident on a focal plane array, $I(m,n)$ where $I(m,n)$ ranges in value from 0 to 1. The normalized energy is then "gamma corrected" (gamma = 2.2) for storage as 8 bit data using the formula

$$x(m,n) = 255 * I(m,n)^{1/2.2}.$$

Unfortunately, your monitor is designed to have a gamma value of 2.0. Find the transformation, $y(m,n) = T\left(x(m,n)\right)$, so that $y(m,n)$ will display properly on your monitor.

4. Consider the following attenuation function

$$g(x,y) = circ(x-1, y-1)$$

   a) Sketch *g*(*x*,*y*).

b) Find its Radon transform

$$p_\theta(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y)\delta(x\cos\theta + y\sin\theta - t)\,dxdy$$

c) Sketch $p_\theta(t)$.

5. The Radon transform $p_\theta(t)$ of an unknown object $g(x,y)$ is given by

$$p_\theta(t) = \delta(t - \cos(\pi/4 - \theta)) + \delta(t - \cos(-\pi/4 - \theta)) + \delta(t - \cos(3\pi/4 - \theta)) + \delta(t - \cos(-3\pi/4 - \theta))$$

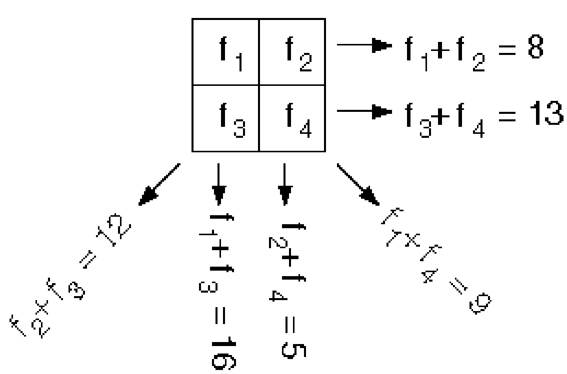
a) Sketch $p_\theta(t)$ for $\theta = 0,\ \pi/4,\ \pi/2,\ 3\pi/4$, and π.
b) Calculate

$$P_\theta(f) = \int_{-\infty}^{\infty} p_\theta(t) e^{-j2\pi f t}\,dt$$

c) Use the Fourier slice theorem to determine $G(u,v)$.
d) Take the inverse 2D continuous-space Fourier transform (CSFT) of $G(u,v)$ to determine $g(x,y)$.
e) Sketch $g(x,y)$.
f) Verify your answer for $g(x,y)$ by showing that it yields the correct Radon transform

$$p_\theta(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y)\delta(x\cos\theta + y\sin\theta - t)\,dxdy$$

6(optional). Consider a discrete tomography problem in which the unknown attenuation function is divided into four cells, and six ray sums are available as shown below:



(Note that the weights are assumed to be unity.) Apply the algebraic reconstruction technique to determine the unknown attenuation values $f_1, \ldots, f_4$.