EE 438 Digital Signal Processing with Applications
Homework #8 due 4/7/99

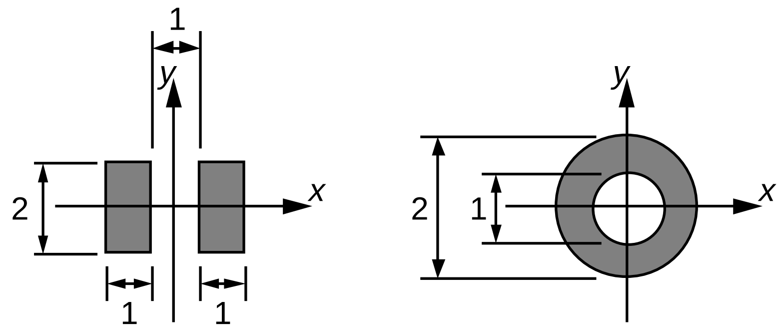
1. For each of the following distributions
   i) compute $p_X(x)$ and $p_Y(y)$
   ii) compute $E[X]$ and $Var[X]$
   iiii) compute $E[XY]$
   iv) show that X and Y are dependent or independent.
   a) $$p_{xy}(x,y) = \begin{cases} 1/4 & |x| \le 1 \text{ and } |y| \le 1 \\ 0 & otherwise \end{cases}$$
   b) $$p_{xy}(x,y) = \begin{cases} 1/2 & |x| + |y| \le 1 \\ 0 & otherwise \end{cases}$$

2. Let X_n have autocorrelation
$$R_x(k) = \rho^{-|k|}\sigma_x^2$$
and let $Y(n)$ be the output of a LTI filter with impulse response
$$h(n) = a^{-n}u(n)$$
   a) Calculate $R_y(k)$
   b) Calculate $S_x(e^{j\omega})$
   c) Calculate $S_y(e^{j\omega})$

3. A zero mean signal $x(n)$ with power spectrum
$$S_x(e^{j\omega}) = 1$$
is passed serially through two systems with transfer functions
$$H_1(z) = \frac{1}{1-0.9z}$$
$$H_2(z) = \frac{1}{1-0.8z}$$
to produce the output $y(n)$.
   a) Compute and plot the power spectrum of $y(n)$ in the range $-\pi$ to π.
   b) Compute and plot the autocorrelation function of $y(n)$.

4. For each of the two functions given below, do the following.
   i) Express $f(x,y)$ in terms of special functions given in class.
   ii) Find its CSFT $F(u,v)$ using transform pairs and properties.
   iii) Sketch $F(u,v)$ in enough detail to show that you know what it looks like.

   Assume that $f(x,y) = 1$ in shaded regions and $f(x,y) = 0$ elsewhere.



5. The 2-D DFT is defined by

$$X(l,k) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} x(m,n)e^{-j2\pi(lm+kn)/N}$$

Show that the 2-D DFT can be computed by computing a 1-D DFT along n

$$\hat{X}(m,k) = \sum_{n=0}^{N-1} x(m,n)e^{-j2\pi kn/N}$$

followed by a 1-D DFT along m

$$X(l,k) = \sum_{n=0}^{N-1} \hat{X}(m,k)e^{-j2\pi lm/N}$$

6. For a real image $f(x,y)$, show that the CSFT $F(u,v) = |F(u,v)|e^{j\angle F(u,v)}$ has the following symmetry:
   a) $|F(u,v)| = |F(-u,-v)|$
   b) $\angle F(u,v) = -\angle F(-u,-v)$

7. Compute the DSFT $X(e^{j\mu}, e^{j\gamma})$ of the following functions for $|a| < 1$
   a. $x(n,m) = a^n u(n)\delta(m)$
   b. $x(n,m) = a^{|n+m|} u(n)u(m)$
   c. $x(n,m) = a^{|n+m|}$ (this problem is modified to result in a simpler solution.)