EE 438 Digital Signal Processing with Applications
Homework #6 due 3/8/99

1. The objective of this problem is to numerically compute the CTFT of the function

   $$x(t) = e^{-t}u(t)$$

   using MatLab and the FFT algorithm. This will be done by sampling the signal

   $$y(n) = x(nT)$$

   and then computing the approximate DTFT $Y(e^{j\omega})$. The CTFT $X(f)$ can then be computed by appropriate scaling of $Y(e^{j\omega})$. (Hint: use the sampling formula.)

   a) Choose the sampling rate f_s so that the aliased signal is less than 5% of its peak value

   $$|X(f_s/2)| \le 0.05|X(0)|$$

   b) Choose the window size L so that the window contains at least 99% of the signals energy.

   $$\int_0^{(L-1)T} |x(t)|^2 dt \ge 0.99 \int_0^{\infty} |x(t)|^2 dt$$

   c) Use the DFT on Matlab (call using the function fft()) to compute and plot an approximate CTFT $X(f)$. **Scale and label the axes correctly**.

2. The objective of this problem is to show that the DFT and inverse DFT formulas are correct. We will first show that

   $$y(n) = \sum_{k=0}^{N-1} e^{j2\pi kn/N} = N \sum_{m=-\infty}^{\infty} \delta(n - mN)$$

   in three steps.

   a) Show that $y(n)$ is periodic with period N.
   b) Show that $y(0) = N$.
   c) Show that $y(n) = 0$ for $n = 1,\ldots,N-1$.
   d) Use direct substitution to show that $\mathrm{DFT}^{-1}\{\mathrm{DFT}\{x(n)\}\} = x(n)$.

3. Consider the following discrete time functions.

   $$x(n) = \Lambda((n-3)/4)$$

   $$y(n) = \delta(n) + n\,u(3-n)u(n)$$

   Compute and plot the periodic convolution of $x(n)$ and $y(n)$ with period:

   a) $N = 7$
   b) $N = 10$
   c) $N = 15$

4. Consider the following length 10 sequences:

n	0	1	2	3	4	5	6	7	8	9
$x_1[n]$	32	16	8	4	2	1	0	0	0	0
$x_2[n]$	1	2	2	3	3	3	4	4	4	4

a. Calculate the aperiodic convolution of $x_1[n]$ and $x_2[n]$.

b. Calculate the periodic (period 10) convolution of $x_1[n]$ and $x_2[n]$.

c. To what length would the sequences need to be padded with zeros so that a portion of their periodic convolution would match the nonzero part of their aperiodic convolution?