EE 438 Digital Signal Processing with Applications
Homework #5 due 3/1/99

1. Consider the ZT

$$H(z) = \frac{z^2 - z}{(z^2 - 1/4)(z - 1/2)}.$$

   a) Find the poles and zeros of this ZT.
   b) Sketch the poles, zeros and the possible ROC's.
   c) For each ROC, determine if the impulse response is causal, right sided, or left sided.
   d) For each ROC, determine if the impulse response is stable or unstable.
   d) For each ROC, compute the corresponding signal $h(n)$.
   e) Find a difference equation which implements this transfer function, and draw its flow diagram.

2. Consider the following difference equation

$$y(n) = ay(n-1) + x(n) - x(n-1).$$

   a) Compute the transfer function $H(z) = \frac{Y(z)}{X(z)}$, and find its poles and zeros.
   b) Compute the impulse response $h(n)$ using a ROC of $|z| > a$. For what values of a is the system stable?
   c) Compute the impulse response $h(n)$ using a ROC of $|z| < a$. For what values of a is the system stable?

3. Consider the causal D-T LTI system described by the following recursive difference equation

$$y(n) = x(n) - x(n-8) + y(n-1)$$

   a) Find the transfer function $H(z)$ for this filter.
   b) Sketch the locations of poles and zeros in the complex z-plane.
   c) For each ROC, find the impulse response $h(n)$ by computing the inverse ZT of $H(z)$.
   d) Is this filter IIR or FIR? Explain your answer.

4. Use the form of the Z-transform sum to show that each of the following is true for $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$. (Remember, the ROC is defined as the set of z such that $\sum_{n=-\infty}^{\infty} \left| x(n) z^{-n} \right| < \infty$.)
   a) If the ROC contains $z = 0$, then the $x(n)$ is anticausal.
   b) If the ROC contains $z = \infty$, then the $x(n)$ is causal.
   c) If z_0 is in the ROC and $x(n)$ is causal, then the ROC includes all $|z| > z_o$.
   d) If z_0 is in the ROC and $x(n)$ is anticausal, then the ROC includes all $|z| < z_o$.

5. Find expressions for the N point DFT's of the following signals.

   a. $x(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n = 1, \ldots, N-1 \end{cases}$

   b. $x(n) = (-1)^n$, $n = 0, \ldots, N-1$

   c. $x(n) = e^{j2\pi nk/N}$, $n = 0, \ldots, N-1$ where k is an integer between 0 and $N-1$.

   d. $x(n) = \cos(2\pi nk/N)$, $n = 0, \ldots, N-1$ where k is an integer between 0 and $N-1$.