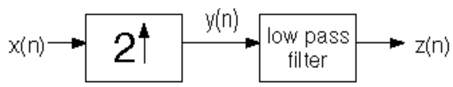
EE 438 Digital Signal Processing with Applications
Homework #4 due 2/22/98

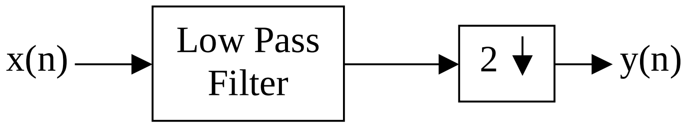
1) The following system shows an interpolator with discrete input $x(n)$. Assume that the low pass filter has frequency response $H(e^{j\omega}) = 2\text{rect}(\omega/\pi)$ for $|\omega| < \pi$.



Compute the output $z(n)$ for the following inputs.

a) $x(n) = \cos(\pi n/4)$
b) $x(n) = \cos(3\pi n/4)$
c) $x(n) = \delta(n)$
d) $x(n) = \text{sinc}(n/8)$
e) $x(n) = \text{sinc}(3n/4)$

2) The following system shows a decimator with discrete input $x(n)$. Assume that the low pass filter has frequency response $H(e^{j\omega}) = \text{rect}(\omega/\pi)$ for $|\omega| < \pi$.



Compute the output $z(n)$ for the following inputs.

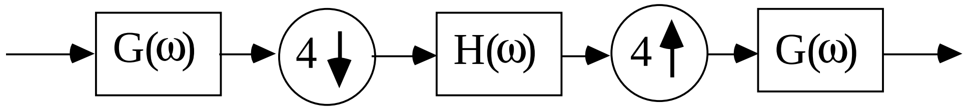
a) $x(n) = \cos(\pi n/4)$
b) $x(n) = \cos(3\pi n/4)$
c) $x(n) = \delta(n)$
d) $x(n) = \text{sinc}(n/8)$
e) $x(n) = \text{sinc}(3n/4)$

3) Consider the digital filter described by the following difference equation

$$y[n] = 0.25(x[n+1] + 2x[n] + x[n-1])$$

a) Find a simple expression for the frequency response $H(\omega)$ of this filter.
b) Sketch the magnitude and phase of $H(\omega)$.

Now consider the following digital system,



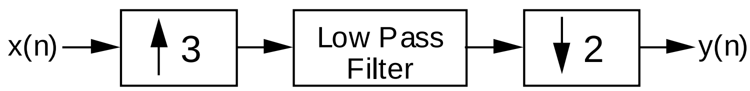
where $H(\omega)$ is the filter from parts a and b and $G(\omega)$ is an ideal low-pass filter with a cutoff frequency of $\pi/4$ rad/sample and unity gain in the passband.

c) Find the overall frequency frequency response $F(\omega)$ for this system

d) Sketch the magnitude and phase of $F(\omega)$.

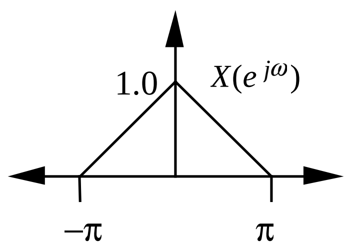
e) Discuss the possible advantages of a system like that shown above compared to directly implementing a digital filter with frequency response $F(\omega)$ as a single stage.

4) Consider the following system where the low pass filter has a cutoff frequency of $\pi/3$.



a) Calculate a simple expression for $Y(e^{j\omega})$ in terms of $X(e^{j\omega})$.

b) Carefully sketch $Y(e^{j\omega})$, assuming that $X(e^{j\omega})$ is given by



c) Compute the number of multiplies required per output sample $y(n)$. Assume that the low pass filter length is not symmetric and of length N. (Hint: Do not count any multiplication by zero, and remember that the decimator only requires every other output from the low pass filter.)

5) For each signal, do the following:

i) Find the ZT (if it exists) and the region of convergence in terms of a and b.

ii) For the given value of a and b, sketch the ROC indicating where poles and zeros occur.

a) $x(n) = e^{jbn}u(n)$; $b = 0.25\pi$

b) $x(n) = a^n u(n)$; $a = 0.5$

c) $x(n) = a^{-|n|}$; $a = 0.5$

d) $x(n) = \cos(bn)u(n)$; $b = 0.1$

e) $x(n) = \dfrac{1}{n!}u(n)$

f) $x(n) = \delta(n) - a(\delta(n-1) + \delta(n+1))$; $a = 0.5$