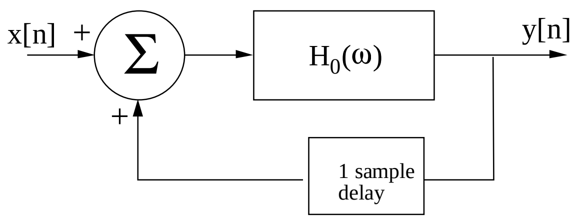
EE 438 Digital Signal Processing with Applications
Homework #3 due 2/8/99

1. A LTI system with frequency response $H(e^{j\omega})$ has input $\cos(\omega_o n)$. Derive a simple expression for the output of the system. Repeat for input $\sin(\omega_o n)$.

2. Consider the system shown below where the filter is described by the difference equation $y[n] = (x[n] - y[n-1])/2$:



   a. Find a difference equation that describes the overall system.

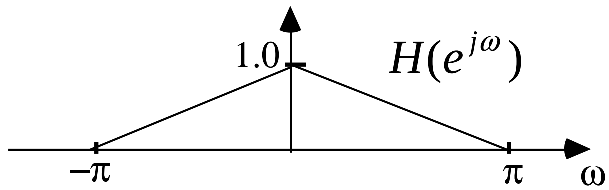
   b. Find an expression for the frequency response $H(\omega)$ of the overall system in terms of $H_0(\omega)$, the frequency response of the filter.

   c. Find the actual frequency response $H(\omega)$ from your answer to part a. and also using your answer to part b. Verify that the two approaches lead to the same answer.

3. Let $y(n)$ be a D-T signal formed by sampling the C-T signal $x(t)$ so that $y(n)$ = $x(nT)$. For each case do the following:
   i) calculate an analytical expression for $Y(e^{j\omega})$
   ii) use Matlab to plot $y(n)$
   iii) use Matlab to plot the magnitude and phase of $Y(e^{j\omega})$ for $\omega \in [-\pi, \pi]$.

   a) $x(t) = \text{sinc}(t),\ T = 1$
   b) $x(t) = \text{sinc}(t - 1/4),\ T = 1$
   c) $x(t) = \text{sinc}(t),\ T = 1/2$
   d) $x(t) = \text{sinc}(t),\ T = 3/2$
   e) $x(t) = \cos(2\pi t)\text{sinc}(t/8), T = 1/4$
   f) $x(t) = \cos(\pi t)\text{sinc}(t/8), T = 1/4$
   g) $x(t) = \cos(2\pi t)\text{sinc}(t/4), T = 1/4$

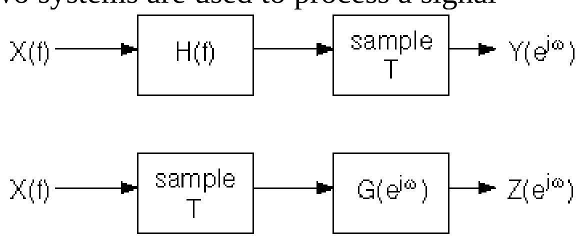
4. The analog signal $x(t) = \sin(2\pi f_0 t)$ is sampled with an ideal sampler at a rate of 8 kHz, filtered with a digital filter having the frequency response $H(e^{j\omega})$ shown below, and then reconstructed as an analog signal $y(t)$ with an impulse generator followed by an ideal low pass filter with a cutoff frequency of 4 kHz.



Find the output $y(t)$ for the following values of f_0.

a. 1 kHz,
b. 4 kHz,
c. 5 kHz.

5. The following two systems are used to process a signal



Assume that the input signal $X(f)$ and the continuous time filter $H(f)$ are both band limited to 1/(2T).

a) Compute analytic expressions for $Y(e^{j\omega})$ and $Z(e^{j\omega})$.
b) Show that if

$$G(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H\left(\frac{\omega - 2\pi k}{2\pi T}\right)$$

then the outputs of the two systems are equal.
c) Explain what (if any) advantages the second system may have.

6. You decide that you must have a programmable time delay for your new band. In order to make the design flexible you decide to sample the signal, process it digitally, and then reconstruct it. You band limit the signal to 20*kHz*, use a sampling rate of 44*kHz*, and reconstruct with an impulse generator followed by an ideal low pass filter of 22*kHz*. Assuming that there is no delay in the sampling and reconstruction process:

a) Find the digital filter $H(e^{j\omega})$ that will produce a pure time delay of d sec. (Hint: Find a digital filter so that the ratio $X_r(f)/X(f) = e^{-j2\pi fd}$.)
b) Plot the phase and magnitude of $H(e^{j\omega})$ over the interval $[0,\ 2\pi]$ for i) $d = 22.727\ \mu\text{sec}$ ii) $d = 35\ \mu\text{sec}$.
c) What is the impulse response $h(n)$ when $d = 22.727\ \mu\text{sec}$.
d) Find the ideal filter $H(e^{j\omega})$ that will produce a pure time delay d when a zero order sample and hold is used for reconstruction.