EE 438 Digital Signal Processing with Applications
Homework #2 due 1/29/99

1. Consider a DT LTI system described by the following equation

$$y(n) = x(n) + 2x(n-1) + 0.5y(n-1).$$

   a. Compute the impulse response $h(n)$ of the system.

   b. Compute the output when $x(n) = u(n)$.

   c. Compute the output when $x(n) = 0.25^n u(n)$.

2. For each of the following C-T signals, compute the CTFT and manually plot the magnitude of the result.

   a) $e^{-t}u(t)$
   b) $e^{j\omega_0 t}$
   c) $\mathrm{rect}(t)e^{j6\pi t}$
   d) $\mathrm{sinc}(t)\cos(2\pi f_0 t)$
   e) $\cos(2\pi f_o t)\mathrm{rect}(\mathrm{t})$

3. For each of the following D-T signals,
   i. Compute the DTFT $X(\omega)$. Simplify your answer as much as possible.
   ii. Sketch the magnitude and phase of $X(\omega)$.

   a) $u(n+N) - u(n-N-1)$
   b) $2^n u(-n)$
   c) $a^n \sin(\omega_o n)u(n) \; |a|<1, \; |\omega_o|<\pi$
   d) $\cos(18\pi n/7)$
   e) $\dfrac{\sin(\pi n/8)}{\pi n}$

4. Let $x(n)$ and $y(n)$ be D-T signals with DTFT's $X(e^{j\omega})$ and $Y(e^{j\omega})$ respectively. Use the formulas for the DTFT and its inverse to compute the DTFT's of the following signals.

   a) $x(n-N)e^{j\omega_0 n}$
   b) $x^*(-n)$
   c) $x(n)y(\mathrm{n})$
   d) $x(n)^2$

5. Consider the filter described by the difference equation

$$y[n] = \frac{1}{4}\{x[n] - 2x[n-1] + x[n-2]\}$$

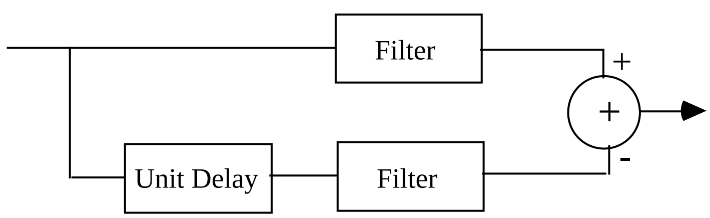
a. Find a simple expression for the frequency response $H(\omega)$
b. Find a simple expression for the magnitude response $|H(\omega)|$
c. Sketch $|H(\omega)|$
d. Find a simple expression for the phase response $\arg\{H(\omega)\}$
e. Sketch $\arg\{H(\omega)\}$

6. For the LTI systems below,

i. find the impulse response,
ii. find an expression for the frequency response (simplify as much as possible),
iii. sketch the magnitude and phase of the frequency response,
iv. describe in general terms the effect that the filter has on a signal.

a. $y[n] = (x[n] + x[n-1])/2$
b. $y[n] = (x[n] - y[n-3])/2$
c. $y[n] = (x[n] - 2x[n-1] + x[n-2])/4$

7. Consider the system shown below where the filter is described by the difference equation $y[n] = (x[n] + x[n-1])/2$:



a. Find a difference equation that describes the overall system.
b. Find an expression for the frequency response $H(\omega)$ of the overall system in terms of $H_0(\omega)$, the frequency response of the filter.
c. Find the actual frequency response $H(\omega)$ from your answer to part a. and also using your answer to part b. Verify that the two approaches lead to the same answer.