EE 438 Digital Signal Processing with Applications
Homework #1 due 1/22/99

1. For each signal below, do the following:

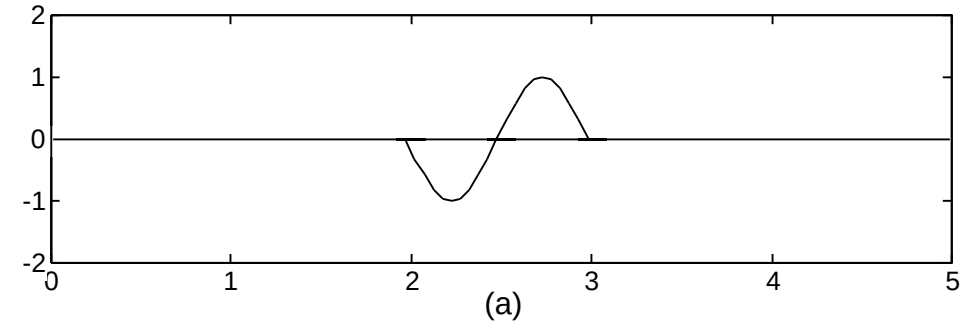
   i. Sketch x(t)
   ii. State whether it is right-sided, left-sided, or two-sided.
   iii. State whether it is causal, anti-causal, or neither.
   iv. Calculate the metrics E_x, P_x, x_{rms}, M_x, A_x, and x_{avg}.

   a. $x(t) = te^{-t/2}u(t)$

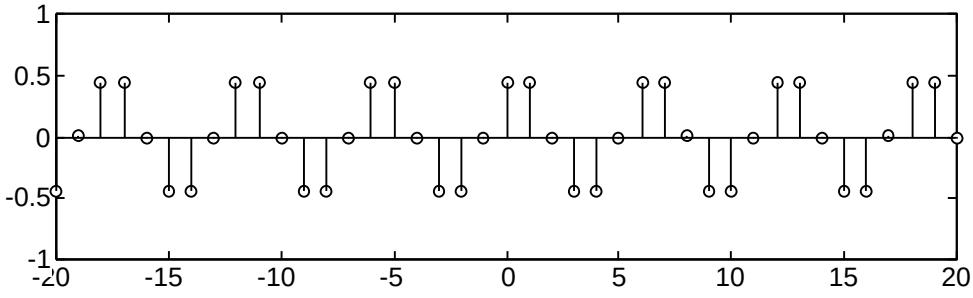
   b. $x(t) = \sum_k (-1)^k \text{ rect}(t/2 - 2k)$

   c. $x[n] = e^{(j-1)pn/2}$

2. Express each signal shown below in terms of standard functions. Note that the signal for part (b) is a sinusoid, and should be expressed as such.



(a)



(b)

3. For each system below, determine whether or not it is:
   i. linear,
   ii. time-invariant,
   iii. causal,

iv. stable,
v. memoryless

For each of the above properties, if you think it holds, prove it. Otherwise, find a counter-example. In addition, find the response to an impulse.

a) $y(t) = \int_{t-1}^{t} x(t)\mathrm{d}t$

b) $y(t) = \mathrm{rect}(x(t))$

c) $y[n] = x[n] + (1/2)y[n-1]$

d) $y[n] = \begin{cases} x[n], & x[n] < |n| \\ |n|, & \text{else} \end{cases}$

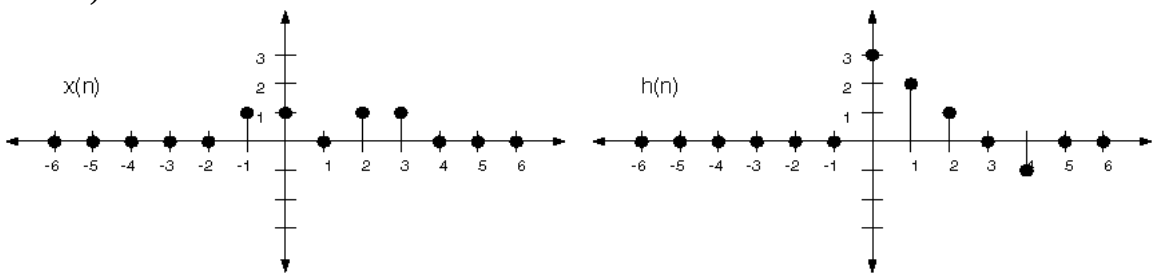
4. A LTI system has input $x(n)$ and impulse response $h(n)$. Compute the output $y(n)$ for each of the following cases (a, $b < 1$).

a) $x(n) = u(n) - u(n-4)$; $h(n) = a^n u(n)$

b) $x(n) = a^n u(n)$; $h(n) = b^n u(n)$ $(a \neq b)$

c) $x(n) = a^n u(n)$; $h(n) = a^n u(n)$

d)



5. Find a general expression for the N roots z_i, $i = 0, 1, \ldots, N-1$ of the following polynomial, where z and w are complex-valued: $z^N - w = 0$. *Hint:* Express both z and w in polar coordinates, and note that angles need only match within a multiple of $2p$ radians. Sketch the roots in the complex plane for the following values of N and w:

a. $N = 4$, $w = -2$,
b. $N = 5$, $w = j$,