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EE 438 DIGITAL SIGNAL PROCESSING WITH APPLICATIONS

Final Exam – Tuesday, May 4, 1999

- You have 110 minutes to complete the following SIX problems.
- It is to your advantage to budget your time so that you can try every problem.
- The examination is closed-book, closed-notes and open mind.
- You must show all work to obtain full credit.
- No calculators are allowed.

Good Luck!

Some useful formulas:

1-D Transforms

$$\text{rect}(t) \overset{CTFT}{\Leftrightarrow} \text{sinc}(f)$$
$$\text{sinc}(t) \overset{CTFT}{\Leftrightarrow} \text{rect}(f)$$
$$e^{-\pi t^2} \overset{CTFT}{\Leftrightarrow} e^{-\pi f^2}$$
$$x(t/T) \overset{CTFT}{\Leftrightarrow} |T|\,X(fT)$$
$$x(t-d) \overset{CTFT}{\Leftrightarrow} X(f)e^{-j2\pi fd}$$
$$x(t)e^{j2\pi f_o} \overset{CTFT}{\Leftrightarrow} X(f-f_o)$$

Sampling

$$Y(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\frac{\omega-2\pi k}{2\pi T}\right)$$
$$S(f) = Y(e^{j2\pi fT})$$

Interpolation and Decimation

$$Z(e^{j\omega}) = Y(e^{jL\omega})$$
$$Z(e^{j\omega}) = \frac{1}{L}\sum_{k=0}^{L-1} Y\left(e^{j(\omega-2\pi k)/L}\right)$$

2-D Transforms

$$\text{rect}(x,y) \overset{CSFT}{\Leftrightarrow} \text{sinc}(u,v)$$
$$\text{circ}(x,y) \overset{CSFT}{\Leftrightarrow} \text{jinc}(u,v)$$
$$\text{circ}(x,y) = \begin{cases} 1 & if\ \sqrt{x^2+y^2} < 1/2 \\ 0 & otherwise \end{cases}$$

Z-Transforms

$$a^n u(n) \Leftrightarrow \frac{1}{1-az^{-1}} \quad ROC = |z| > a$$
$$-a^n u(-1-n) \Leftrightarrow \frac{1}{1-az^{-1}} \quad ROC = |z| < a$$

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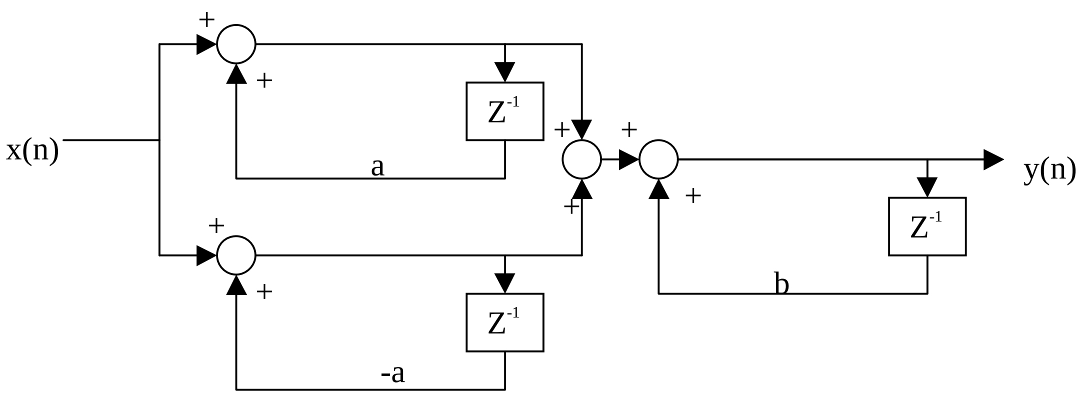
Problem 1. (33 points)

Consider the following discrete time system.



a) Calculate the transfer function $H(z)$ for the following system.

b) Calculate the difference equation for the system.

c) Calculate the poles of the system.

d) Assuming that the system is causal, when is the difference equation stable?

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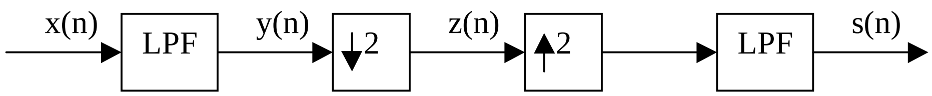
Problem 2. (33 points)
Consider the following discrete time system where $x(n) = \text{sinc}(n/4)$ and the low pass filters have a cut-off frequency of $\pi/2$ and a gain of 1.



a) Calculate $X(e^{j\omega})$.
b) Calculate $y(n)$ and $Y(e^{j\omega})$.
c) Calculate $z(n)$ and $Z(e^{j\omega})$.
d) Calculate $s(n)$ and $S(e^{j\omega})$.

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Problem 3. (33 points)

Compute the 8-point DFT of the following signals.

a) $x(n) = \begin{cases} 1 & \text{for } 0 \le n \le 2 \\ 0 & \text{for } 3 \le n \le 7 \end{cases}$

b) $y(n) = e^{j2\pi n/4}$ for $0 \le n \le 7$

c) $z(n) = \sum_{l=0}^{7} x(l) e^{j2\pi(n-l)/4}$

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Problem 4. (33 points)
Let the 1-D discrete time function $h(n)$ have the DTFT $H(e^{j\omega}) = rect(\omega/\pi)$ for $|\omega| < \pi$, and let the 2-D discrete time function $f(m,n)$ have the DSFT
$F(e^{j\mu}, e^{j\nu}) = rect(\mu/\pi, \nu/\pi) - rect(2\mu/\pi, 2\nu/\pi)$ for $|\mu| < \pi$ and $|\nu| < \pi$.

a) Calculate $h(n)$.
b) Is $f(m,n)$ a separable function? Justify your answer.
c) Calculate $f(m,n)$.
d) Calculate $\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f(m,n)$.

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Problem 5. (34 points)
Let $h(m,n) = (0.9)^{m+n} u(m)\, u(n)$.

a) Calculate the 2-D Z-transform $H(z_1, z_2)$.
b) Calculate and sketch $\left|H(e^{j\mu}, e^{j\nu})\right|$ the magnitude of the 2-D DSFT for $|\mu| < \pi$ and $|\nu| < \pi$.
c) Write a difference equation for the system with impulse response $h(m,n)$.

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Problem 6. (34 points)
Consider the function $f(x, y) = rect(x/2, y) + rect(x, y/2)$.

a) Sketch the function $f(x, y)$.
b) Let $g_\theta(t)$ be the projections of $f(x, y)$. Calculate $g_\theta(t)$ for $\theta = 0$.
c) Consider the shifted function $f(x-10, y-10)$. Calculate the $p_\theta(t)$, the projections of $f(x-10, y-10)$, in terms of the function $g_\theta(t)$.
(Assume that you know $g_\theta(t)$ for all θ and t).

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