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EE 438 DIGITAL SIGNAL PROCESSING WITH APPLICATIONS

Exam #3 – Spring 1999

Friday, April 16, 1999

- You have 50 minutes to complete the following THREE problems.
- It is to your advantage to budget your time so that you can try every problem.
- The examination is closed-book, closed-notes.
- You must show all work to obtain full credit.
- No calculators are allowed.

Some useful formulas:

1-D Transforms

$$\text{rect}(t) \overset{CTFT}{\Leftrightarrow} \text{sinc}(f)$$

$$\text{sinc}(t) \overset{CTFT}{\Leftrightarrow} \text{rect}(f)$$

$$e^{-\pi t^2} \overset{CTFT}{\Leftrightarrow} e^{-\pi f^2}$$

$$x(t/T) \overset{CTFT}{\Leftrightarrow} |T| X(fT)$$

$$x(t-d) \overset{CTFT}{\Leftrightarrow} X(f)e^{-j2\pi fd}$$

$$x(t)e^{j2\pi f_o t} \overset{CTFT}{\Leftrightarrow} X(f-f_o)$$

2-D Transforms

$$\text{rect}(x,y) \overset{CSFT}{\Leftrightarrow} \text{sinc}(u,v)$$

$$\text{circ}(x,y) \overset{CSFT}{\Leftrightarrow} \text{jinc}(u,v)$$

$$\text{circ}(x,y) = \begin{cases} 1 & \text{if } \sqrt{x^2+y^2} < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

Sampling

$$Y(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right)$$

$$S(f) = Y(e^{j2\pi fT})$$

Z-Transforms

$$a^n u(n) \Leftrightarrow \frac{1}{1-az^{-1}} \quad ROC = |z| > a$$

$$-a^n u(-1-n) \Leftrightarrow \frac{1}{1-az^{-1}} \quad ROC = |z| < a$$

Interpolation and Decimation

$$Z(e^{j\omega}) = Y(e^{jL\omega})$$

$$Z(e^{j\omega}) = \frac{1}{L}\sum_{k=0}^{L-1} Y\left(e^{j(\omega - 2\pi k)/L}\right)$$

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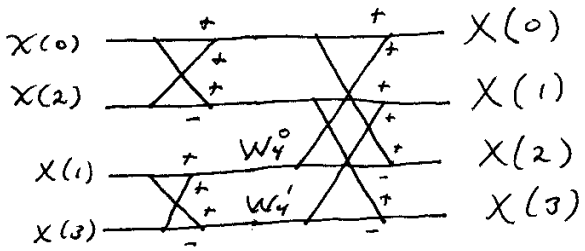
Problem 1.

a) (10 points) Draw a flow diagram for a 4-point FFT. Make sure to clearly label the inputs and outputs and the coefficients in your flow diagram.

b) (10 points) Count the number of multiplies required for your 4-point DFT. Do not count multiplies by 1,-1,j, or –j.

c) (10 points) Draw a flow diagram for a 10-point DFT by using two 5-point DFT's. Draw each 5-point DFT as a single box with 5 inputs and 5 output. Make sure to clearly label the inputs and outputs and the coefficients in your flow diagram.

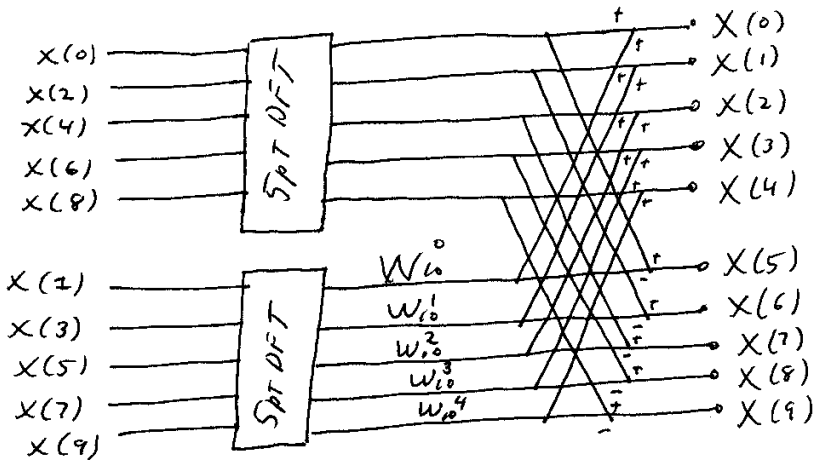
a)



$W_4^0 = 1 \quad W_4^1 = -j$

b) Number of Complex Multiplies = 0

c)



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Problem 2.

Let x_n be a sequence of i.i.d. N(0, σ^2) random variables that are input into a system with impulse response $h(n) = \delta(n) - \delta(n-1)$ to produce the output y_n.

a) (20 points) Calculate and plot the autocorrelation and power spectrum for x_n.

b) (20 points) Calculate and plot the autocorrelation and power spectrum for y_n.

a) $\mu = E[x_n] = 0$ $\quad R_x(m) = E[(x_n - \mu)(x_{n+m} - \mu)]$

$$= E[x_n x_{n+m}]$$

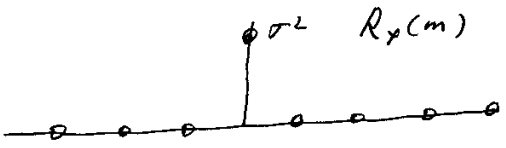
$$= \begin{cases} E[x_n]\,E[x_{n+m}] & \text{for } m \neq 0 \\ E[x_n^2] & \text{for } m = 0 \end{cases}$$

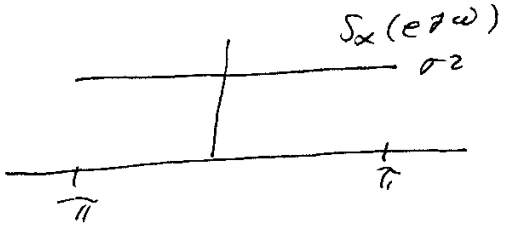
$$= \begin{cases} 0 & \text{for } m \neq 0 \\ \sigma^2 & \text{for } m = 0 \end{cases}$$

$$= \sigma^2 \delta(m)$$

$$S_x(e^{j\omega}) = DTFT\{R_x(m)\}$$

$$= \sigma^2$$





b) $R_y(m) = R_x(m) * h(n) * h(-n)$

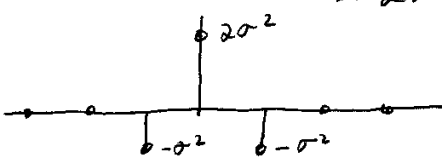
$$h(n) * h(-n) = -\delta(n+1) + 2\delta(n) - \delta(n-1)$$

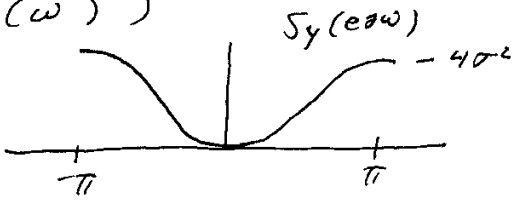
$$R_y(m) = \sigma^2(-\delta(n+1) + 2\delta(n) - \delta(n-1))$$

$$S_y(e^{j\omega}) = \sigma^2(2 - e^{j\omega} - e^{-j\omega})$$

$$= \sigma^2(2 - 2\cos(\omega))$$

$$= 2\sigma^2(1 - \cos(\omega))$$





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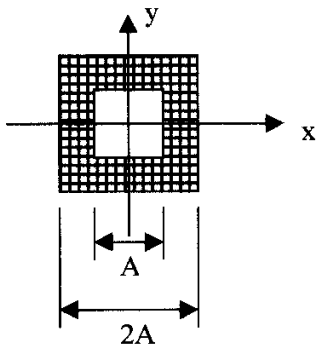
Problem 3.

Consider the 2-D function $g(x,y)$ where the hash marks indicate the value 1.



a) (10 points) Give an expression for $g(x,y)$ and its CSFT $G(u,v)$.

b) (10 points) Plot the function $G(u,0)$. Label all important values on the plot including the value at $u=0$.

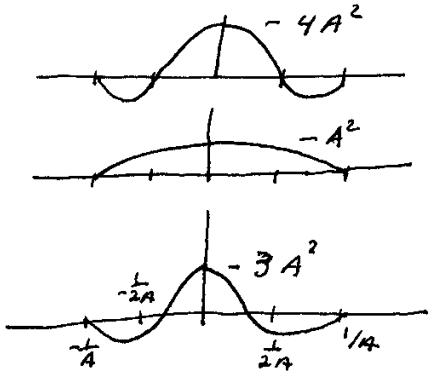
c) (10 points) Calculate the CSFT of the function $f(x,y) = rep_{4A,4A}\{g(x,y)\}$.

a) $g(x,y) = rect\left(\frac{x}{2A}, \frac{y}{2A}\right) - rect\left(\frac{x}{A}, \frac{y}{A}\right)$

$G(u,v) = 4A^2 sinc(2Au, 2Av) - A^2 sinc(Au, Av)$

$= A^2 (4 sinc(2Au, 2Av) - sinc(Au, Av))$

b)



c) $F(u,v) = \frac{1}{(4A)^2} comb\left(\frac{1}{4A}, \frac{1}{4A}\right)\left\{A^2 (4 sinc(2Au, 2Av) - sinc(Au, Av)\right\}$