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EE 438 DIGITAL SIGNAL PROCESSING WITH APPLICATIONS

Exam #3 – Spring 1999

Friday, April 16, 1999

- You have 50 minutes to complete the following THREE problems.
- It is to your advantage to budget your time so that you can try every problem.
- The examination is closed-book, closed-notes.
- You must show all work to obtain full credit.
- No calculators are allowed.

Some useful formulas:

1-D Transforms

$$\text{rect}(t) \overset{CTFT}{\Leftrightarrow} \text{sinc}(f)$$

$$\text{sinc}(t) \overset{CTFT}{\Leftrightarrow} \text{rect}(f)$$

$$e^{-\pi t^2} \overset{CTFT}{\Leftrightarrow} e^{-\pi f^2}$$

$$x(t/T) \overset{CTFT}{\Leftrightarrow} |T|X(fT)$$

$$x(t-d) \overset{CTFT}{\Leftrightarrow} X(f)e^{-j2\pi fd}$$

$$x(t)e^{j2\pi f_o t} \overset{CTFT}{\Leftrightarrow} X(f-f_o)$$

2-D Transforms

$$\text{rect}(x,y) \overset{CSFT}{\Leftrightarrow} \text{sinc}(u,v)$$

$$\text{circ}(x,y) \overset{CSFT}{\Leftrightarrow} \text{jinc}(u,v)$$

$$\text{circ}(x,y) = \begin{cases} 1 & \text{if } \sqrt{x^2+y^2} < 1/2 \\ 0 & \text{otherwise} \end{cases}$$

Sampling

$$Y(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right)$$

$$S(f) = Y(e^{j2\pi fT})$$

Z-Transforms

$$a^n u(n) \Leftrightarrow \frac{1}{1-az^{-1}} \quad ROC = |z| > a$$

$$-a^n u(-1-n) \Leftrightarrow \frac{1}{1-az^{-1}} \quad ROC = |z| < a$$

Interpolation and Decimation

$$Z(e^{j\omega}) = Y(e^{jL\omega})$$

$$Z(e^{j\omega}) = \frac{1}{L}\sum_{k=0}^{L-1} Y\left(e^{j(\omega - 2\pi k)/L}\right)$$

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Problem 1.

a) (10 points) Draw a flow diagram for a 4-point FFT. Make sure to clearly label the inputs and outputs and the coefficients in your flow diagram.
b) (10 points) Count the number of multiplies required for your 4-point DFT. Do not count multiplies by 1,-1,j, or –j.
c) (10 points) Draw a flow diagram for a 10-point DFT by using two 5-point DFT's. Draw each 5-point DFT as a single box with 5 inputs and 5 output. Make sure to clearly label the inputs and outputs and the coefficients in your flow diagram.

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Problem 2.

Let x_n be a sequence of i.i.d. N(0, σ^2) random variables that are input into a system with impulse response $h(n) = \delta(n) - \delta(n-1)$ to produce the output y_n.

a) (20 points) Calculate and plot the autocorrelation and power spectrum for x_n.

b) (20 points) Calculate and plot the autocorrelation and power spectrum for y_n.

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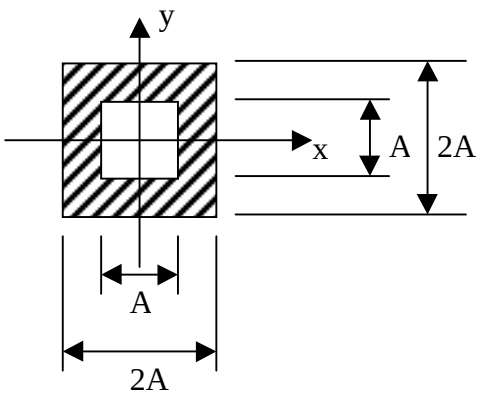
Problem 3.

Consider the 2-D function $g(x, y)$ where the hash marks indicate the value 1.



a) (10 points) Give an expression for $g(x, y)$ and its CSFT $G(u, v)$.

b) (10 points) Plot the function $G(u,0)$ for $|u| < 1/A$. Label all important values on the plot including the value at $u = 0$.

c) (10 points) Calculate the CSFT of the function $f(x, y) = rep_{4A,4A}\{g(x, y)\}$.

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