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EE 438 DIGITAL SIGNAL PROCESSING WITH APPLICATIONS
Exam #2 – Spring 1999
Friday, March 12, 1999

- You have 50 minutes to complete the following THREE problems.
- It is to your advantage to budget your time so that you can try every problem.
- The examination is closed-book, closed-notes.
- You must show all work to obtain full credit.
- No calculators are allowed.

Some useful formulas:

1-D Transforms

$$\text{rect}(t) \overset{CTFT}{\Longleftrightarrow} \text{sinc}(f)$$

$$\text{sinc}(t) \overset{CTFT}{\Longleftrightarrow} \text{rect}(f)$$

$$e^{-\pi t^2} \overset{CTFT}{\Longleftrightarrow} e^{-\pi f^2}$$

$$x(t/T) \overset{CTFT}{\Longleftrightarrow} |T| X(fT)$$

$$x(t-d) \overset{CTFT}{\Longleftrightarrow} X(f)e^{-j2\pi fd}$$

$$x(t)e^{j2\pi f_o} \overset{CTFT}{\Longleftrightarrow} X(f-f_o)$$

Sampling

$$Y(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right)$$

$$S(f) = Y(e^{j2\pi fT})$$

Interpolation and Decimation

$$Z(e^{j\omega}) = Y(e^{jL\omega})$$

$$Z(e^{j\omega}) = \frac{1}{L}\sum_{k=0}^{L-1} Y\left(e^{j(\omega - 2\pi k)/L}\right)$$

2-D Transforms

$$\text{rect}(x,y) \overset{CSFT}{\Longleftrightarrow} \text{sinc}(u,v)$$

$$\text{circ}(x,y) \overset{CSFT}{\Longleftrightarrow} \text{jinc}(u,v)$$

$$\text{circ}(x,y) = \begin{cases} 1 & if \ \sqrt{x^2 + y^2} < 1/2 \\ 0 & otherwise \end{cases}$$

Z-Transforms

$$a^n u(n) \Longleftrightarrow \frac{1}{1 - az^{-1}} \quad ROC = |z| > a$$

$$-a^n u(-1-n) \Longleftrightarrow \frac{1}{1 - az^{-1}} \quad ROC = |z| < a$$

Problem 1. (33 points)

Consider the following discrete time system.
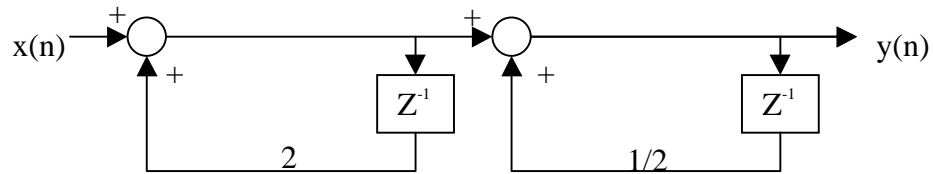
a) Calculate its causal impulse response.

b) Calculate its stable impulse response.

c) Calculate its left sided impulse response.

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Problem 2. (33 points)

Consider the following two discrete-time functions
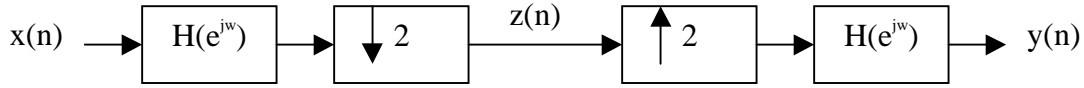$$x(n) = u(n) - u(n-8)$$
$$y(n) = \cos(2\pi n / N)$$

For the following problems, show all work.

a) Calculate the DTFT of $x(n)$.

b) Calculate the 16-point DFT of $x(n)$.

c) Calculate the 16-point DFT of $y(n)$.

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Problem 3. (34 points)

Consider the following discrete-time system.

$$x(n) \longrightarrow \boxed{H(e^{jw})} \longrightarrow \boxed{\downarrow 2} \xrightarrow{z(n)} \boxed{\uparrow 2} \longrightarrow \boxed{H(e^{jw})} \longrightarrow y(n)$$

For the following problems, assume that $X(e^{j\omega}) = \Lambda(\omega/\pi)$ for $|\omega| < \pi$.

a) Sketch $X(e^{j\omega})$ for $|\omega| < \pi$.

b) Calculate and sketch $Z(e^{j\omega})$ and $Y(e^{j\omega})$ when

$$H(e^{j\omega}) = \begin{cases} 1 & |\omega| < \pi/2 \\ 0 & \pi/2 \leq |\omega| < \pi \end{cases}$$

c) Calculate and sketch $Z(e^{j\omega})$ and $Y(e^{j\omega})$ when

$$H(e^{j\omega}) = \begin{cases} 0 & |\omega| < \pi/2 \\ 1 & \pi/2 \leq |\omega| < \pi \end{cases}$$

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