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EE 438 DIGITAL SIGNAL PROCESSING WITH APPLICATIONS

Exam #1 – Spring 1999

Friday, February 12, 1999

- You have 50 minutes to complete the following FIVE problems.
- It is to your advantage to budget your time so that you can try every problem.
- The examination is closed-book, closed-notes.
- You must show all work to obtain full credit.
- No calculators are allowed.

Some useful formulas:

1-D Transforms

$$\mathrm{rect}(t) \overset{CTFT}{\Leftrightarrow} \mathrm{sinc}(f)$$

$$\mathrm{sinc}(t) \overset{CTFT}{\Leftrightarrow} \mathrm{rect}(f)$$

$$e^{-\pi t^2} \overset{CTFT}{\Leftrightarrow} e^{-\pi f^2}$$

$$x(t/T) \overset{CTFT}{\Leftrightarrow} |T|X(fT)$$

$$x(t-d) \overset{CTFT}{\Leftrightarrow} X(f)e^{-j2\pi fd}$$

$$x(t)e^{j2\pi f_o} \overset{CTFT}{\Leftrightarrow} X(f-f_o)$$

2-D Transforms

$$\mathrm{rect}(x,y) \overset{CSFT}{\Leftrightarrow} \mathrm{sinc}(u,v)$$

$$\mathrm{circ}(x,y) \overset{CSFT}{\Leftrightarrow} \mathrm{jinc}(u,v)$$

$$\mathrm{circ}(x,y) = \begin{cases} 1 & if \sqrt{x^2+y^2} < 1/2 \\ 0 & otherwise \end{cases}$$

Sampling

$$Y(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right)$$

$$S(f) = Y(e^{j2\pi fT})$$

Interpolation and Decimation

$$Z(e^{j\omega}) = Y(e^{jL\omega})$$

$$Z(e^{j\omega}) = \frac{1}{L}\sum_{k=0}^{L-1} Y\left(e^{j(\omega-2\pi k)/L}\right)$$

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Problem 1. (20 points)
Let $H(e^{j\omega}) = \mathrm{rect}(\omega/\omega_o)$ for $|\omega| < \pi$ where $0 < \omega_o < 2\pi$. Compute $h(n)$, the inverse DTFT of $H(e^{j\omega})$.

$$\frac{\omega_o}{2\pi} \operatorname{sinc}\left(\frac{\omega_o}{2\pi} t\right) \overset{CTFT}{\Longleftrightarrow} \operatorname{rect}\left(\frac{2\pi}{\omega_o} f\right)$$

<u>Sample at $T=1$</u>

$$h(n) = \frac{\omega_o}{2\pi} \operatorname{sinc}\left(\frac{\omega_o}{2\pi} n\right)$$

$$H(e^{j\omega}) = \frac{1}{1} \sum_{K=-\infty}^{\infty} \operatorname{rect}\left(\frac{2\pi}{\omega_o}\left(\frac{\omega - 2\pi K}{2\pi\, 1}\right)\right)$$

$$= \sum_{K=-\infty}^{\infty} \operatorname{rect}\left(\frac{\omega - 2\pi K}{\omega_o}\right)$$

For $0 \le \omega_o < 2\pi$, we then have

$$H(e^{j\omega}) = \operatorname{rect}(\omega/\omega_o) \text{ for } |\omega| < \pi$$

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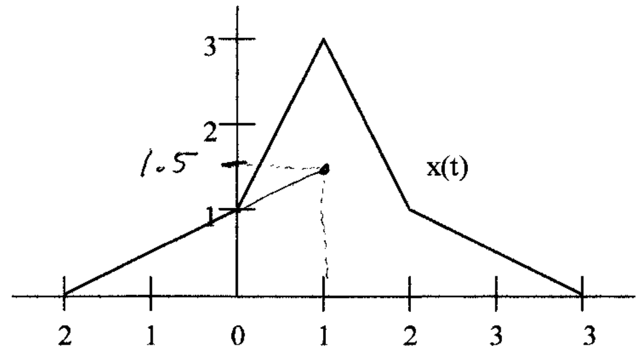
Problem 2. (20 points)
Compute the CTFT of the following signal $x(t)$.



$$x(t) = 1.5\Lambda\left(\frac{t-1}{3}\right) + 1.5\Lambda(t-1)$$

$$X(f) = 4.5\,\mathrm{sinc}^2(3f)e^{-j2\pi f} + 1.5\,\mathrm{sinc}^2(f)e^{-j2\pi f}$$

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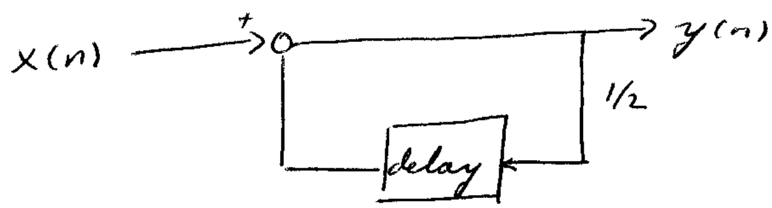
Problem 3 (20 points)
Let $h(n) = (-1/2)^n u(n)$ be the impulse response of a system. Find a difference equation for this system, and compute $H(e^{j\omega})$.



$$y(n) = x(n) - \frac{1}{2} y(n-1)$$

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (-1/2)^n u(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \left(-\frac{1}{2} e^{-j\omega}\right)^n$$

$$= \frac{1}{1 + \frac{1}{2} e^{-j\omega}}$$

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Problem 4 (20 points)

Let $h(n) = \text{sinc}(n/2)$.

a) Compute the area for $h(n)$.

b) Compute the energy for $h(n)$.

a) $\text{sinc}(t/2) \overset{CTFT}{\Longleftrightarrow} 2\,\text{rect}(2f)$

$\text{sinc}(n/2) \overset{DTFT}{\Longleftrightarrow} 2\,\text{rect}(\omega/\pi)$ for $|\omega| < \pi$

$$\text{area} = \sum_{n=-\infty}^{\infty} \text{sinc}(n/2)$$

$$= \sum_{n=-\infty}^{\infty} \text{sinc}(n/2)\, e^{-j0n}$$

$$= H(e^{j0})$$

$$= 2\,\text{rect}(0/\pi)$$

$$= 2$$

b) $\text{sinc}^2(t/2) \overset{CTFT}{\Longleftrightarrow} 2\Lambda(2f)$

$\text{sinc}^2(n/2) \overset{DTFT}{\Longleftrightarrow} 2\Lambda(\omega/\pi)$ for $|\omega| < \pi$

$$\text{Energy} = \sum_{n=-\infty}^{\infty} \text{sinc}^2(n/2)$$

$$= \sum_{n=-\infty}^{\infty} \text{sinc}^2(n/2)\, e^{-j0n}$$

$$= 2\Lambda(0/\pi)$$

$$= 2$$

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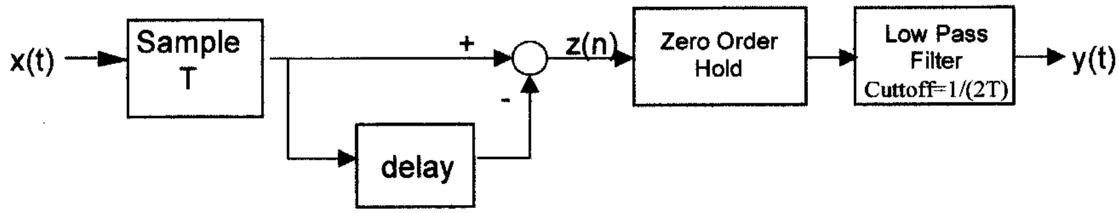
Problem 5 (20 points)
Consider the following system for sampling and reconstructing signals. Assume that $x(t)$ is band limited to $|f| < 1/(2T)$.



a) Compute $Z(e^{j\omega})$ in terms of the CTFT of $x(n)$ for $|\omega| < \pi$.

b) Compute the frequency response $H(f)$ for the complete system.

$$Z(e^{j\omega}) = (1-e^{j\omega})\frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\frac{\omega-2\pi k}{2\pi T}\right)$$

$$= e^{-j\omega/2}\frac{2j}{T}\frac{1}{2j}(e^{j\omega/2}-e^{-j\omega/2})\,X\left(\frac{\omega}{2\pi T}\right) \quad \text{for } |\omega|<\pi$$

$$= \frac{2j}{T}e^{-j\omega/2}\sin(\omega/2)\,X\left(\frac{\omega}{2\pi T}\right) \quad \text{for } |\omega|<\pi$$

$$Y(f) = T\,\mathrm{sinc}(fT)\left[\frac{2j}{T}e^{-j\omega/2}\sin(\omega/2)X\left(\frac{\omega}{2\pi T}\right)\right]\Bigg|_{\omega=2\pi fT}$$

$$= T\,\mathrm{sinc}(fT)\frac{2j}{\cancel{T}}e^{-j\pi fT}\sin(\pi fT)\,X(f)$$

$$H(f) = \frac{Y(f)}{X(f)} = 2j\,e^{-j\pi fT}\,\mathrm{sinc}(fT)\sin(\pi fT)$$