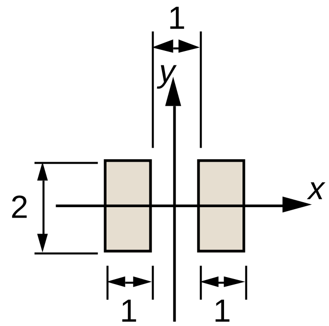
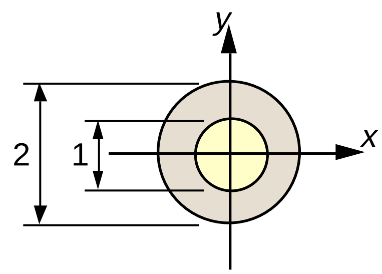
EE 438 Digital Signal Processing with Applications
Homework #9 12/4/95

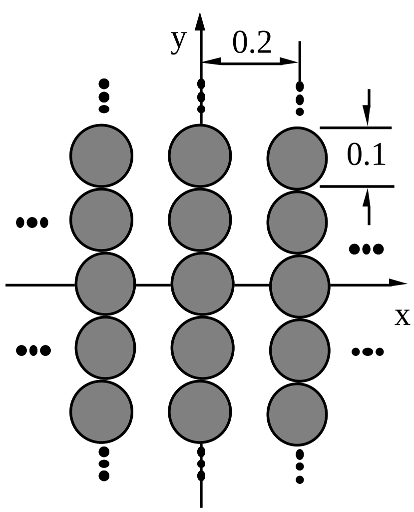
1) For each of the two functions given below, do the following.
   i) Express $f(x,y)$ in terms of special functions given in class.
   ii) Find its CSFT $F(u,v)$ using transform pairs and properties.
   iii) Sketch $F(u,v)$ in enough detail to show that you know what it looks like.
   Assume that $f(x,y) = 1$ in shaded regions and $f(x,y) = 0$ elsewhere.

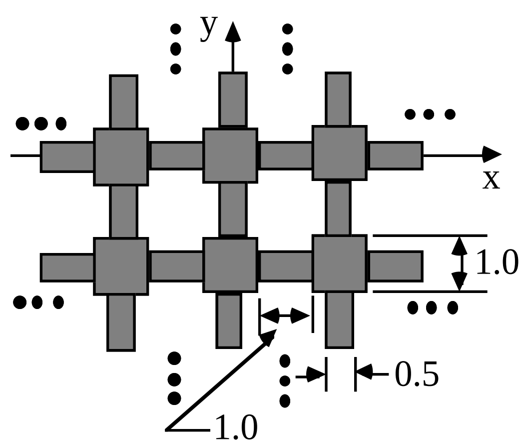




2) For each of the two functions given below, do the following.
   i) Express $f(x,y)$ in terms of special functions given in class.
   ii) Find its CSFT $F(u,v)$ using transform pairs and properties.
   iii) Sketch $F(u,v)$ in enough detail to show that you know what it looks like.

Assume that $f(x,y) = 1$ in shaded regions and $f(x,y) = 0$ elsewhere





(c)

3) Let $g(x,y)$ be a continuous parameter image with energy band limited to the frequencies $|u| < \pi / T$ and $|v| < \pi / T$. Let $s(m,n)$ be an ideal discrete parameter image formed by sampling $g(x,y)$.

$$s(m,n) = g(mT, nT)$$

Furthermore, let $w(m,n)$ be the discrete parameter image obtained by a CCD array.

$$w(m,n)=\int_{(m-\frac{1}{2})T}^{(m+\frac{1}{2})T}\int_{(m-\frac{1}{2})T}^{(m+\frac{1}{2})T} g(x,y)dxdy$$

a) Compute $S\left(e^{j\mu},e^{j\nu}\right)$.

b) Compute $W\left(e^{j\mu},e^{j\nu}\right)$.

c) Compute $\dfrac{W\left(e^{j\mu},e^{j\nu}\right)}{S\left(e^{j\mu},e^{j\nu}\right)}$ for $|\mu|<\pi$ and $|\nu|<\pi$

4) A monochrome image $f(m,n)$ has histogram

$$h_f(b)=\begin{cases}100 & \text{for } 100\le b\le 199\\ 0 & \text{otherwise}\end{cases}.$$

The new image $g(m,n)$ is then formed by the transformation $g(m,n)=T\left[f(m,n)\right]$

a) Computed the number of pixels in $f(m,n)$ which are less then B for any value of B.

b) Plot the function computed in part a) for $0\le B\le 255$

c) Determine the piecewise linear function $T[.]$ such that the histogram of $g(m,n)$ is given by

$$h_g(b)=100 \quad \text{for } 0\le b\le 255$$

d) Repeat parts a) and b) for the histogram $h_g(b)$.

5) A monochrome image is obtained by measuring the normalized energy incident on a focal plane array, $I(m,n)$ where $I(m,n)$ ranges in value from 0 to 1. The normalized energy is then "gamma corrected" (gamma = 2.2) for storage as 8 bit data using the formula

$$x(m,n)=255*I(m,n)^{1/2.2}.$$

Unfortunately, your monitor is designed to have a gamma value of 2.0. Find the transformation, $y(m,n)=T\left(x(m,n)\right)$, so that $y(m,n)$ will display properly on your monitor.

6) An image $x(m,n)$ is filtered to generate the output image $y(m,n)$. The impulse response of the filter is given by

$$h(m,n)=\delta(m,n)-h_{avg}(m,n)$$

where

$$h_{avg}(m,n)=\begin{cases}1/9 & |m|<=1 \text{ and } |n|<=1\\ 0 & \text{otherwise}\end{cases}.$$

a) Sketch the 2-D impulse response $h(m,n)$.

b) Compute the 2-D frequency response $H(e^{-j\mu},e^{-j\upsilon})$.

c) Compute $y(m,n)$ when $x(m,n)=u(m)$ where $u(m)$ is a step function.

7) A two dimensional filter has input $x(m,n)$, output $y(m,n)$ and impulse response

$$h(m,n)=0.5^m 0.25^n u(m)u(n)$$

a) Compute the frequency response $H(e^{j\mu},e^{j\nu})$.

b) Find a recursive difference equation for the system.

c) Compute y when $x(m,n) = u(m)$ and sketch your result.