EE 438 Digital Signal Processing with Applications
Homework #8 due 11/10/95

1. An analog speech signal has a bandwidth of 3kHz and can be modeled as a stationary random process with a marginal probability density that is Gaussian with mean zero and varianced 1. You are asked to design a system to sample, quantize and store the speech signal on a computer.
   a) Compute the Nyquist samplin rate for the data.
   b) Choose a quantizer range so that the probability of overload is approximately 0.02. (**Hint:** You will need to use a table for the cumulative distribution function of a Gaussian random variable.)
   c) Assuming that the quantization noise may be modeled as a random variable uniformly distributed between $[-\Delta/2, \Delta/2]$, and a B bit quantizer is used, determine an expression for the average signal-to-noise power ratio in dB defined as

$$SNR = 10\log_{10}\frac{E\{(y(n))^2\}}{E\{(\varepsilon(n))^2\}}$$

   where $y(n)$ are the samples of the speech signal before quantization, and $\varepsilon(n)$ are the quantization errors.
   d) Find the minimum number of bits so that SNR ≥ 30.
   e) Using the result of d) and a) find the number of bits/second required to digitize the speech data.

2. Compute the DSFT $X(e^{j\mu}, e^{j\gamma})$ of the following functions
   a. $x(n,m) = a^n u(n)\delta(m)$
   b. $x(n,m) = a^{|n+m|}u(m)$

3. A 2-D linear shift invariant discrete space system has an impulse response of

$$h(m,n) = \delta(m,n) - 0.1\{u(m+1) - u(m-2)\}\{u(n+1) - u(n-2)\}$$

   a) Find the 2-D frequency response of this system by computing the 2-D DSFT

$$H(e^{j\mu}, e^{j\upsilon}) = \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} h(m,n)e^{-j(\mu m+\upsilon n)}$$

   (Hint: use the fact that $h(m,n) - \delta(m,n)$ is a separable function.)

b) Use Matlab to compute the approximate DSFT of $h(m,n)$ on the region $[-\pi,\pi]\times[-\pi,\pi]$. Plot the magnitude of the DSFT using the "mesh" command.

c) Next find the inverse filter $g(m,n)$ which will restore an image filtered by $h(m,n)$. Use the formula

$$g(m,n) = F^{-1}\{1/H(e^{j\mu},e^{j\nu})\}$$

and the result of part b) to compute $g(m,n)$. Plot your result using mesh.

4. The 2-D DFT is defined by

$$X(l,k) = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} x(m,n)e^{-j2\pi(lm+kn)/N}$$

Show that the 2-D DFT can be computed by computing a 1-D DFT along n

$$\hat{X}(m,k) = \sum_{n=0}^{N-1} x(m,n)e^{-j2\pi kn/N}$$

followed by a 1-D DFT along m

$$X(l,k) = \sum_{n=0}^{N-1} \hat{X}(m,k)e^{-j2\pi lm/N}$$

5. For a real image $f(x,y)$, show that the CSFT $F(u,v) = |F(u,v)|e^{j\angle F(u,v)}$ has the following symmetry:

a) $|F(u,v)| = |F(-u,-v)|$

b) $\angle F(u,v) = -\angle F(-u,-v)$

6. Consider the following attenuation function

$$g(x,y) = \mathrm{circ}(x,y)$$

a) Sketch $g(x,y)$.

b) Find its Radon transform

$$p_\theta(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\delta(x\cos\theta + y\sin\theta - t)dxdy$$

c) Sketch $p_\theta(t)$.

d) Why doesn't $p_\theta(t)$ depend on θ?

7. The Radon transform $p_\theta(t)$ of an unknown object $g(x,y)$ is given by

$$p_\theta(t) = \delta(t-\cos(\pi/4-\theta)) + \delta(t-\cos(-\pi/4-\theta)) + \delta(t-\cos(3\pi/4-\theta)) + \delta(t-\cos(-3\pi/4-\theta))$$

a) Sketch $p_\theta(t)$ for $\theta = 0,\ \pi/4,\ \pi/2,\ 3\pi/4$, and π.

b) Calculate

$$P_\theta(f) = \int_{-\infty}^{\infty} p_\theta(t) e^{-j2\pi ft} dt$$

c) Use the Fourier slice theorem to determine $G(u,v)$.

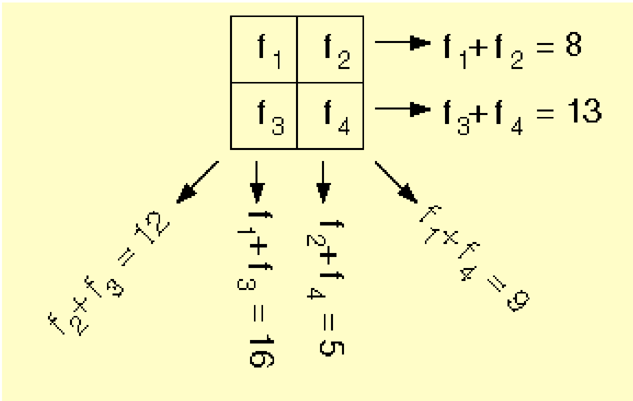
d) Take the inverse 2D continuous-space Fourier transform (CSFT) of $G(u,v)$ to determine $g(x,y)$.

e) Sketch $g(x,y)$.

f) Verify your answer for $g(x,y)$ by showing that it yields the correct Radon transform

$$p_\theta(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y)\delta(x\cos\theta + y\sin\theta - t)dxdy$$

8. Consider a discrete tomography problem in which the unknown attenuation function is divided into four cells, and six ray sums are available as shown below:



(Note that the weights are assumed to be unity.) Apply the algebraic reconstruction technique to determine the unknown attenuation values $f_1, \ldots, f_4$.