EE 438 Digital Signal Processing with Applications
Homework #6 due 10/13/95

1. The objective of this problem is to numerically compute the CTFT of the function

$$x(t) = e^{-t}u(t)$$

using MatLab and the FFT algorithm. This will be done by sampling the signal

$$y(n) = x(nT)$$

and then computing the approximate DTFT $Y(e^{j\omega})$. The CTFT $X(f)$ can then be computed by appropriate scaling of $Y(e^{j\omega})$. (Hint: use the sampling formula.)

a) Choose the sampling rate f_s so that the aliased signal is less than 5% of its peak value

$$|X(f_s/2)| \leq 0.05\,|X(0)|$$

b) Choose the window size L so that the window contains at least 99% of the signals energy.

$$\int_0^{(L-1)T} |x(t)|^2\,dt \geq 0.99\int_0^{\infty} |x(t)|^2\,dt$$

c) Use the DFT on Matlab (call using the function fft()) to compute and plot an approximate CTFT $X(f)$. **Scale and label the axes correctly**.

2. Consider the following discrete time functions.

$$x(n) = \Lambda((n-3)/4)$$
$$y(n) = \delta(n) + n\,u(3-n)u(n)$$

Compute and plot the periodic convolution of $x(n)$ and $y(n)$ with period:

a) $N = 7$
b) $N = 10$
c) $N = 15$

3. Draw the flow diagram for a 4 point decimation in time FFT. Determine the number of complex multiplies required to implement the algorithm. Do not count multiplies +1, -1, j or -j.

4. The signal $x(t)$ is known to have the following three properties:

$$\int_{-\infty}^{\infty} |x(\tau)|^2 \, d\tau = 1$$
$$|x(t)| < 2e^{-t}u(t)$$
$$|X(f)| < 2e^{-|f|}$$

a) Choose the sampling period T which guarantees that the aliased energy is less then 0.1%.

b) Choose the window length L which (approximately) guarantees that the energy loss due to windowing is less then 0.1%.

c) Choose N so that the DFT values represent frequency samples separated by less than 0.1 Hz.