EE 438 Digital Signal Processing with Applications
Homework #5 due 10/6/95

1. Consider the ZT

$$X(z) = \frac{z+1}{z^3 - 2z^2 + \frac{3}{2}z - \frac{1}{2}}$$

Sketch the 3 different ROC's that are possible for this ZT; and for each ROC, find the corresponding signal x[n].

2. Consider a causal D-T LTI system described by the following *recursive* difference equation

$$y[n] = \frac{1}{5}\{x[n] - x[n-5]\} + y[n-1]$$

a. Find the transfer function $H(z)$ for this filter.

b. Sketch the locations of poles and zeros in the complex *z*-plane.

c. Find the impulse response $h[n]$ for this filter by computing the inverse ZT of $H(z)$. Is it of finite or infinite duration?

3. Consider the following difference equation

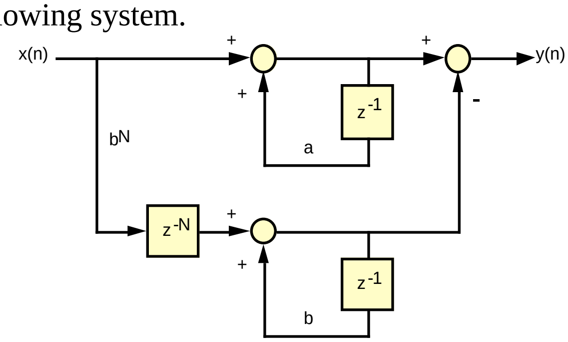
$$y(n) = ay(n-1) + x(n) - x(n-1).$$

a) Compute the transfer function $H(z) = \frac{Y(z)}{X(z)}$, and find its poles and zeros.

b) Compute the impulse response $h(n)$ using a ROC of $|z| > a$. For what values of a is the system stable?

c) Compute the impulse response $h(n)$ using a ROC of $|z| < a$. For what values of a is the system stable?

4. Consider the following system.



a) Calculate the transfer function of the system $H(z)$.

b) Calculate **all** the zeros and poles of the system when $b \neq a$.

c) Calculate **all** the zeros and poles of the system when $b = a$.

d) Calculate the impulse response of the system $h(n)$ when $a = b$. Show that $h(n)$ is FIR.

5. Show the following equation is valid. (Consider two cases: when n is a multiple of N, and when n is not a multiple of N.)

$$\sum_{k=0}^{N-1} e^{j2\pi kn/N} = N \sum_{k=-\infty}^{\infty} \delta(n-kN)$$

For each of the following signals,

i) use this relation to compute the N point DFT's of the following signals

ii) Sketch the result for $N = 8,\ l = 2$.

a) $x(n) = 1,\ n = 0, \ldots, N-1$

b) $x(n) = e^{2\pi jnl/N},\ n = 0, \ldots, N-1$

c) $x(n) = (-1)^n,\ n = 0, \ldots, N-1$

6. Consider the following length 10 sequences:

n	0	1	2	3	4	5	6	7	8	9
$x_1[n]$	32	16	8	4	2	1	0	0	0	0
$x_2[n]$	1	2	2	3	3	3	4	4	4	4

a. Calculate the aperiodic convolution of $x_1[n]$ and $x_2[n]$.

b. Calculate the periodic (period 10) convolution of $x_1[n]$ and $x_2[n]$.

c. To what length would the sequences need to be padded with zeros so that a portion of their periodic convolution would match the nonzero part of their aperiodic convolution?