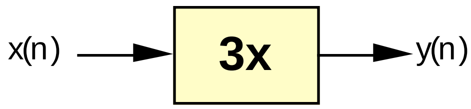
# EE 438 Digital Signal Processing with Applications
# Homework #4 due 9/29/95

1. Consider the system shown below which repeats each sample in the sequence three times, as illustrated



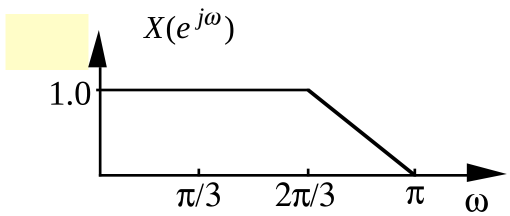

0,-1,1,2,... 0,0,0,-1,-1,-1,1,1,1,2,2,2,..

   a. Determine whether or not this system is
      i. linear
      ii. time-invariant
      iii. causal
      iv. BIBO stable

      For each property, either prove that it holds or provide a counterexample.

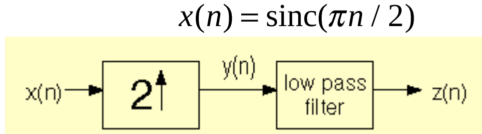
   b. For an arbitrary input $x(n)$ with DTFT $X(e^{j\omega})$, find a simple expression for the DTFT $Y(e^{j\omega})$ in terms of $X(e^{j\omega})$. *Hint:* Represent the system as an upsampler followed by a filter.

   c. Carefully sketch $Y(e^{j\omega})$, assuming that $X(e^{j\omega})$ is given by




2. The following system shows an interpolator with discrete input

$$x(n) = \text{sinc}(\pi n / 2)$$

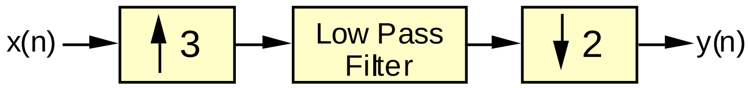



The discrete time low pass filter is ideal and has a cut off frequency of $\omega = \pi / 2$.

a) Calculate expressions for the DTFT's $X(e^{j\omega})$, $Y(e^{j\omega})$, and $Z(e^{j\omega})$. Plot these three functions.

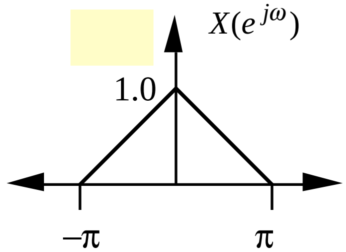
b) Calculate expressions for the discrete time signals $y(n)$ and $z(n)$. Plot these two functions.

3. Consider the following system where the low pass filter has a cutoff frequency of $\pi/3$.




a) Calculate a simple expression for $Y(e^{j\omega})$ in terms of $X(e^{j\omega})$.

b) Carefully sketch $Y(e^{j\omega})$, assuming that $X(e^{j\omega})$ is given by




c) Compute the number of multiplies required per output sample $y(n)$. Assume that the low pass filter length is not symetric and of length $N$. (Hint: Do not count any multiplication by zero, and remember that the decimator only requires every other output from the low pass filter.)

4. For each signal, do the following:
   i) Find the ZT, if it exists.
   ii) Sketch the ROC indicating where poles and zeros occur.

a) $x(n) = 0.5^n u(n) + 2^n u(-n)$

b) $x(n) = \delta(n+1) + 2\delta(n) + \delta(n-1)$.

c) $x(n) = \frac{2^n}{n!} u(n)$

d) $x(n) = e^{j\pi n/4} u(n)$

5. Consider a DT LTI system described by the following *nonrecursive* difference equation (moving average filter)

$$y[n] = \frac{1}{5}\{x[n] + x[n-1] + x[n-2] + x[n-3] + x[n-4]\}$$

a. Find the impulse response $h[n]$ for this filter. Is it of finite or infinite duration?

b. Find the transfer function $H(z)$ for this filter.

c. Sketch the locations of poles and zeros in the complex $z$-plane.

*Hint:* To factor $H(z)$, use the geometric series and the results of Problem 5 on Assignment No. 1.