EE 438 Digital Signal Processing with Applications
Homework #3 due 9/15/95

1. Let $y(n)$ be a D-T signal formed by sampling the C-T signal $x(t)$ so that $y(n) = x(nT)$. For each case do the following:

   i) calculate an analytical expression for $Y(e^{j\omega})$
   ii) use Matlab to plot $y(n)$
   iii) use Matlab to plot the magnitude and phase of $Y(e^{j\omega})$ for $\omega \in [-\pi, \pi]$.

   a) $x(t) = (\text{sinc}(t))^2,\ T = 1$
   b) $x(t) = (\text{sinc}(t - 1/4))^2,\ T = 1$
   c) $x(t) = (\text{sinc}(t))^2,\ T = 1/2$
   d) $x(t) = (\text{sinc}(t))^2,\ T = 3/2$
   e) $x(t) = \cos(2\pi t)\text{rect}(t/(8\pi)),\ T = 1/8$
   f) $x(t) = \cos(2\pi t)\text{rect}(t/(8\pi)),\ T = 9/8$
   g) $x(t) = \cos(2\pi t)\text{rect}(t/(8\pi)),\ T = 17/8$

2. The analog signal $x(t) = \cos(2\pi f_0 t)$ is sampled with an ideal sampler at a rate of 8 kHz, filtered with a digital filter having the impulse response

$$h(n) = \left(\delta(n) + \delta(n-1)\right)/2,$$

   and then reconstructed as an analog signal $y(t)$ with an impulse generator followed by an ideal low pass filter with a cutoff frequency of 4 kHz.

   Find the output $y(t)$ for the following values of f_0.

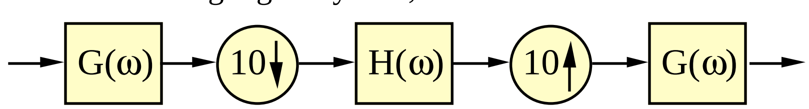
   a. 1 kHz,
   b. 4 kHz,
   c. 5 kHz.

3. Consider the digital filter described by the following difference equation

$$y[n] = \left(x[n] + x[n-1] + x[n-2]\right)/3$$

   a. Find a simple expression for the frequency response $H(\omega)$ of this filter.
   b. Sketch the magnitude of $H(\omega)$.

   Now consider the following digital system,



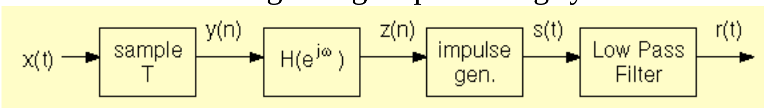
   where $H(\omega)$ is the filter from parts a and b and $G(\omega)$ is an ideal low-pass filter with a cutoff frequency of $\pi/10$ rad/sample and unity gain in the passband.

   c. Find the overall frequency frequency response $F(\omega)$ for this system.

d. Sketch the magnitude of $F(\omega)$.

e. Discuss the possible advantages of a system like that shown above compared to directly implementing a digital filter with frequency response $F(\omega)$ as a single stage.

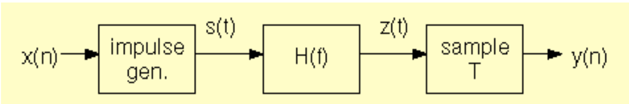
4. The diagram below shows a digital signal processing system.



Assume that the signal $x(t)$ is band limited to half the sampling frequency $f_s = 1/T$. Also assume that the low pass filter is ideal with cut off frequency $f_s/2$.

a) Compute $Z(e^{j\omega})$ in terms of $X(f)$ and $H(e^{j\omega})$.
b) Compute $R(f)$ in terms of $X(f)$ and $H(e^{j\omega})$.
c) Compute the impulse response of the digital filter $h(n)$ so that the complete system produces a delay of $T/2$.
d) Is the complete system linear? Is it time invariant? Explain your answer.

5. The following system processes a digital signal by converting it to a continuous time signal, filtering it, and converting it back to discrete time. The sampling rate is $T = 1/f_s$, and the filter $H(f)$ is band limited to $|f| < f_s/2$.



Compute expressions for each of the following in terms of $X(e^{j\omega})$ and $H(f)$.

a) $S(f)$
b) $Z(f)$
c) $Y(e^{j\omega})$
d) You are told that the input/output behavior of the system should be $\frac{Y(e^{j\omega})}{X(e^{j\omega})} = G(e^{j\omega})$. Choose $H(f)$ in terms of $G(e^{j\omega})$ to achieve this behavior.

6. A real A/D converter does not sample at a single instant. Instead, it averages over a small window. Mathematically, the sampled value $y(n)$ of an analog signal $x(t)$ is given by

$$y(n) = \int_{-\infty}^{\infty} \omega(t) x(t + nT) dt$$

where $\omega(t)$ is a small window. Assume that $X(f)$ is band limited to $|f| < 1/(2T)$.

a) Show that $y(n) = v(nT)$ where $v(t) = \omega(-t) * x(t)$.
b) Calculate $Y(e^{j\omega})$ in terms of $W(f)$ and $X(f)$.
c) Find a digital filter $H(e^{j\omega})$ which will correct the distortion of the averaging window. (Hint: $H(e^{j\omega})Y(e^{j\omega})$ should be the DTFT of the ideal sampled signal $x(nT)$.)