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EE 438 DIGITAL SIGNAL PROCESSING WITH APPLICATIONS

Exam #1 – Fall 1995

Friday, September 22, 1995

- You have 50 minutes to complete the following FOUR problems.
- It is to your advantage to budget your time so that you can try every problem.
- The examination is closed-book, closed-notes and open mind.
- You must show all work to obtain full credit.
- No calculators are allowed.

Some useful formulas:

1-D Transforms

$$\text{rect}(t) \overset{CTFT}{\Leftrightarrow} \text{sinc}(f)$$

$$\text{sinc}(t) \overset{CTFT}{\Leftrightarrow} \text{rect}(f)$$

$$e^{-\pi t^2} \overset{CTFT}{\Leftrightarrow} e^{-\pi f^2}$$

$$x(t/T) \overset{CTFT}{\Leftrightarrow} |T|\, X(fT)$$

$$x(t-d) \overset{CTFT}{\Leftrightarrow} X(f)e^{-j2\pi fd}$$

$$x(t)e^{j2\pi f_o} \overset{CTFT}{\Leftrightarrow} X(f-f_o)$$

2-D Transforms

$$\text{rect}(x,y) \overset{CSFT}{\Leftrightarrow} \text{sinc}(u,v)$$

$$\text{circ}(x,y) \overset{CSFT}{\Leftrightarrow} \text{jinc}(u,v)$$

$$\text{circ}(x,y) = \begin{cases} 1 & if\ \sqrt{x^2+y^2} < 1/2 \\ 0 & otherwise \end{cases}$$

Sampling

$$Y(e^{j\omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(\frac{\omega - 2\pi k}{2\pi T}\right)$$

$$S(f) = Y(e^{j2\pi fT})$$

Interpolation and Decimation

$$Z(e^{j\omega}) = Y(e^{jL\omega})$$

$$Z(e^{j\omega}) = \frac{1}{L}\sum_{k=0}^{L-1} Y\left(e^{j(\omega-2\pi k)/L}\right)$$

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Problem 1. (20 points)

Consider the following systems with input $x(n)$ and output $y(n)$. For each system, either prove that it is linear, or give a counter example.

a) $y(n) = x(n) + 1$

b) $y(n) = x(n)e^{-n} n!$

c) $y(n) = \left(\sum_{k=-\infty}^{\infty} \left(x(k) \right)^2 \right)^{1/2}$

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Problem 2. (25 points)

For parts a) and b) compute the CTFT, and for part c) and d) compute the DTFT. You may use the transform pairs shown on the cover page. Show all work and simplify your answers.

a) $x(t) = \text{rect}(t-3)$

b) $x(t) = \frac{1}{2}\left(\delta(t+1) + \delta(t-1)\right)$

c) $x(n) = (1/2)^n u(n)$

d) $x(n) = u(n) - u(n-N)$

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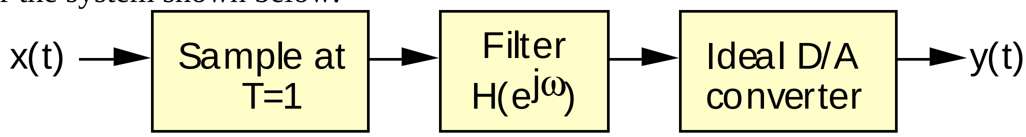
Problem 3. (25 points)
Consider a signal $h(n)$ with the DTFT $H(e^{j\omega}) = \text{rect}(\omega / \pi)e^{-jd\omega}$ for $|\omega| < \pi$ where d is a constant. For each of the following, you must justify your answer by showing all intermediate steps.

a) Schetch the magnitude and phase of $H(e^{j\omega})$ for $|\omega| < 2\pi$

b) Compute the DTFT of $a\,\text{sinc}\big(a(n - b)\big)$ for $0 < a \le 1$. Use this result to compute $h(n)$.

Consider the system shown below.



c) Express $Y(f)$ in terms of $X(f)$ where $X(f)$ is bandlimited to $|f| < 1/2$.

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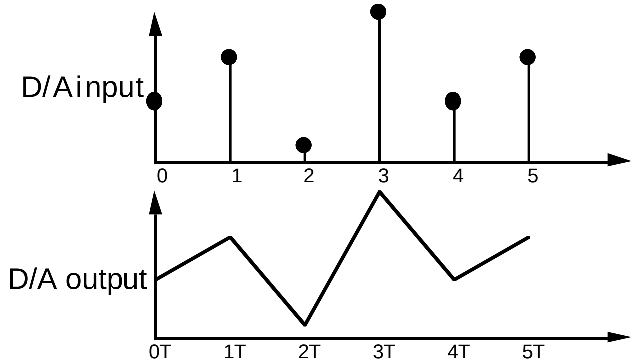
Problem 4. (30 points)

Consider a D/A converter with the behavior illustrated in the figure below. At the sample times (i.e. 0, T, 2T, 3T, ...), the output is equal to the discrete time sample. However, between sample times, the signal varies linearly.



a) Find a function $p(t)$ so that $x_r(t) = \sum_{n=-\infty}^{\infty} p(t-nT)z(n)$ where $z(n)$ is the discrete time input, and $x_r(t)$ is the continuous time output. Justify your answer.

b) Express the signal $X_r(f)$ in terms of $Z(e^{j\omega})$

c) Determine a discrete time filter $H(e^{j\omega})$ which will compensate for the non-ideal behavior of this D/A converter.