Short - Time Synthesis

(Filter Bank Summation Method)

(Show subband coder structure)

STDTFT simply breaks signal into narrowband blocks, and shifts each block down to baseband. Note that we are in time domain!! Can we put signal back together again by simply shifting each block back to where it was, and then summing?

Recall

$$S_r(n) = \sum_k s(k)\, h(n-k)\, e^{-j\frac{2\pi k r}{N}}$$

r ↑ index of subband

$$= \sum_k s(n-k)\, h(k)\, e^{-j\frac{2\pi (n-k) r}{N}}$$

$$= \Big\{ \underbrace{\sum_k s(n-k)\, h_r(k)}_{\text{NB BPF}} \Big\} \underbrace{e^{-j\frac{2\pi n r}{N}}}_{\text{downshift}}$$

where

$$h_r(k) = h(k)\, e^{j\frac{2\pi k r}{N}}$$

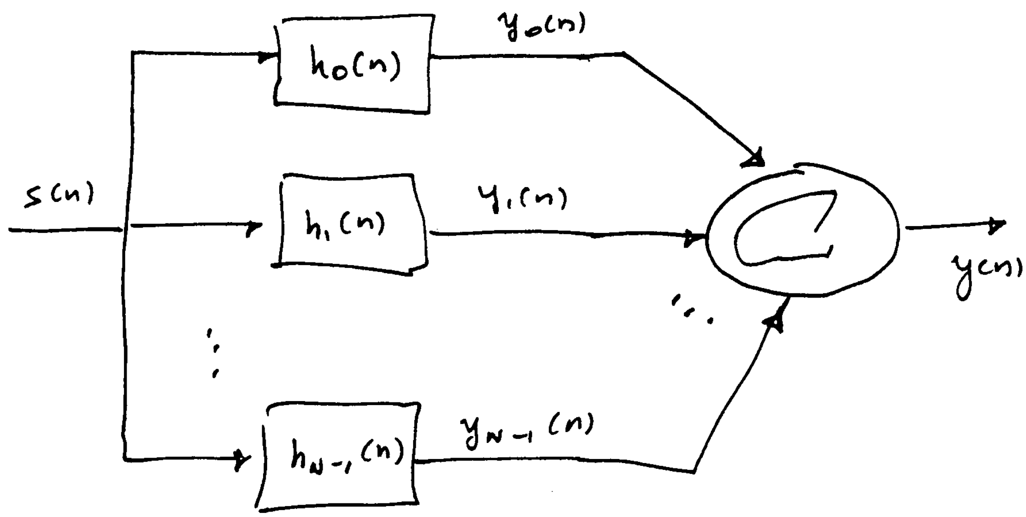
This is just unit sample response of a bandpass filter.

Define

$$y_r(n) = \underbrace{e^{j\frac{2\pi n r}{N}}}_{\text{upshift}} S_r(n)$$

$$= \sum_n s(n-k)\, h_r(k)$$

Now consider following system



Lets find relation between $y(n)$ and $s(n)$

$$y(n) = \sum_{l=0}^{N-1} y_l(n)$$

$\therefore$ $$Y(e^{jw}) = \sum_{l=0}^{N-1} Y_l(e^{jw})$$

$$= \sum_{l=0}^{N-1} S(e^{jw}) H_l(e^{jw})$$

$\therefore$ overall frequency response is

$$H(e^{j\omega}) = \sum_{\ell=0}^{N-1} H_\ell(e^{j\omega})$$

Now

$$H_\ell(e^{j\omega}) = H\left(e^{j\left(\omega - \frac{2\pi \ell}{N}\right)}\right)$$

look at down-sampler followed by upsampler

$\therefore$ in order for $y(n) = s(n)$,

We must have

$$\sum_{\ell=0}^{N-1} H\left(e^{j\left(\omega - \frac{2\pi \ell}{N}\right)}\right) \equiv 1 \quad \forall \; \omega$$

$h(n) \rightarrow \downarrow D \rightarrow \uparrow D \rightarrow \tilde{h}(n)$

$\Rightarrow \tilde{h}(n) = \delta(n)$

What does this imply about $h(n)$?

downsampling $Y(e^{j\omega}) = \frac{1}{D} \sum_{k=0}^{D-1} X\left(e^{j(\omega + 2\pi k)/D}\right)$

upsampling $Y(e^{j\omega}) = X(e^{j\omega D})$

Consider

$$p(n) = N \sum_{k=-\infty}^{\infty} \delta(n - kN)$$

$$P(e^{j\omega}) = N \sum_{k=-\infty}^{\infty} e^{-j\omega k N} = 2\pi \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$$

(derive from rep & comb) (see following page)

Let $\tilde{h}(n) = p(n)\, h(n)$

$$\text{rep}_1[\delta(t)] \xleftrightarrow{\text{CTFT}} \frac{1}{1}\text{comb}_1[1]$$

$\therefore$ $$\sum_k \delta(t-k) \xleftrightarrow{\text{CTFT}} \sum_k \delta(f-k)$$

Now $$\mathcal{F}\left\{\sum_k \delta(t-k)\right\} = \sum_k e^{-j2\pi fk}$$

so $$\sum_k e^{-j2\pi fk} = \sum_k \delta(f-k)$$

Let $\omega N = 2\pi f$

$$\sum_k e^{-j\omega Nk} = \sum_k \delta\left(\frac{\omega N}{2\pi} - k\right)$$

but $$\delta(a\omega - b) = \left|\frac{1}{a}\right| \delta\left(\omega - \frac{b}{a}\right)$$

$\therefore$ $$\sum_k e^{-j\omega Nk} = \frac{2\pi}{N} \sum_k \delta\left(\omega - \frac{2\pi k}{N}\right)$$

or $$\boxed{N \sum_k e^{-j\omega Nk} = 2\pi \sum_k \delta\left(\omega - \frac{2\pi k}{N}\right)}$$

$$\Rightarrow \quad \tilde{H}(e^{j\omega}) = \frac{1}{2\pi}\int_{0}^{2\pi} P(e^{j\mu}) H(e^{j(\omega-\mu)})\, d\mu$$

$$= \sum_{k=0}^{N-1} H(e^{j\omega - \frac{2\pi k}{N}})$$

$\Leftarrow$ Note change in limits

So condition becomes

$$\tilde{h}(n) \equiv \delta(n)$$

i.e. $h(n) = \begin{cases} \frac{1}{N}, & n = 0 \\ 0, & n = kN \quad k \neq 0 \end{cases}$

<u>example:</u>

$$H(e^{j\omega}) = \text{rect}\left(\frac{\omega}{2\pi/N}\right)$$

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \text{rect}\left(\frac{\omega}{2\pi/N}\right) e^{+jn\omega}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/N}^{\frac{\pi}{N}} e^{jn\omega}\, d\omega$$

$$= \frac{1}{2\pi}\,\frac{1}{jn}\left(e^{jn\pi/N} - e^{-jn\pi/N}\right)$$

$$= \frac{1}{N}\left(\frac{N}{\pi n}\right)\sin(\pi n/N)$$

$$= \frac{1}{N}\,\text{sinc}(n/N)$$