Evaluation of

$$S(e^{j\omega}, n) = \sum_k s(k)\, h(n-k)\, e^{-j\omega k}$$

Let $\omega_r = \frac{2\pi r}{N}$, $r = 0, 1, \ldots, N-1$

$$S_r(n) = S(e^{j\omega}, n)\Big|_{\omega=\omega_r}$$

$\angle(r, n)$

actual limits slide along with n

Want to make this look like DFT

$$= \sum_k s(k)\, h(n-k)\, e^{-j\frac{2\pi k r}{N}}$$

Now let $\ell = k - n \Rightarrow k = \ell + n$

slide data forward over the window

$$S_r(n) = \sum_\ell s(\ell+n)\, h(-\ell)\, e^{-j\frac{2\pi}{N} r(\ell+n)}$$

actual limits fixed

illustrate graphically

$$= e^{-j\frac{2\pi}{N} rn} \sum_\ell s(\ell+n)\, h(-\ell)\, e^{-j\frac{2\pi}{N} r\ell}$$

Now let $\ell = \underbrace{m}_{\text{which block}} N + \underbrace{k}_{\text{pos. within block}}$ $\quad -\infty < m < \infty$

$k = 0, 1, \ldots, N-1$

also has row-col interpretation

break sum up into N-length chunks

$$= e^{-j\frac{2\pi}{N} rn} \sum_m \sum_{k=0}^{N-1} s(mN+k+n)\, h(-mN-k) \times e^{-j\frac{2\pi}{N} r(mN+k)}$$

consider n fixed

Let $\tilde{s}(k,n) = \sum_m s(n+k+mN)\, h(-k-mN)$

$$k = 0, 1, \ldots, N-1$$

$$S_r(n) = e^{-j\frac{2\pi}{N}rn} \cdot \sum_{k=0}^{N-1} \tilde{s}(k,n)\, e^{-j\frac{2\pi rk}{N}}$$

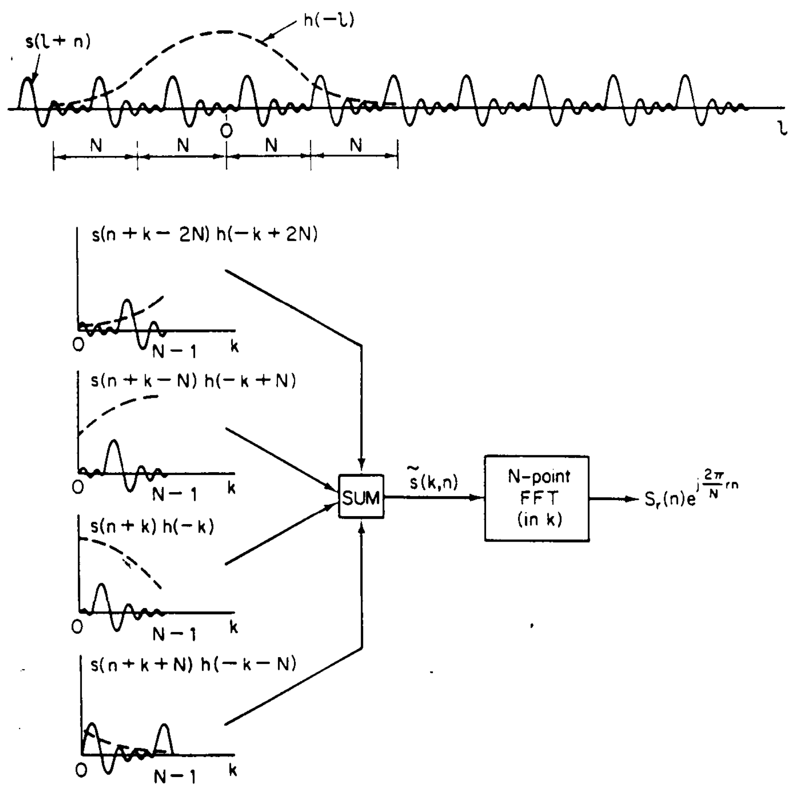

FIGURE 3-13. Evaluation of the short-time Fourier transform using the FFT algorithm.

What is involved here?

Fix n (must repeat everything below for each n)

1. Position data sequence under window
2. Multiply by window
3. Cut up into N-length sequences and sum together
4. Take N-pt DFT
5. Multiply by complex exponential

If window length is L, computation is $\underbrace{L}_{\text{multiply by window}} + \underbrace{\frac{L}{N}}_{\text{number of blocks}} \cdot \underbrace{N}_{\text{sum up the blocks}} + \underbrace{N \log_2 N}_{\text{DFT}} + \underbrace{N}_{\text{phasor mult.}}$

(Note that some operations are real.)

If we let $L = KN$, then have $(2K + \log_2 N + 1)N$

Convolution-based approach:

Identity $\quad rk = \frac{r^2}{2} + \frac{k^2}{2} - \frac{(r-k)^2}{2}$

Substitute into:

$$S_r(n) = \sum_k s(k)\, h(n-k)\, e^{-j\frac{2\pi k r}{N}}$$

$$= \sum_k s(k)\, h(n-k)\, e^{-j\frac{\pi r^2}{N}}\, e^{-j\frac{\pi k^2}{N}}\, e^{j\frac{\pi (r-k)^2}{N}}$$

$$= e^{-j\frac{\pi r^2}{N}} \left[g_n(k) * e^{j\frac{\pi k^2}{N}} \right]$$

$$g_n(k) = s(k)\, h(n-k)\, e^{-j\frac{\pi k^2}{N}}$$

What is involved?

Fix n

1. Position data sequence under window
2. Multiply by window & complex exponential
3. Perform convolution
4. Multiply by complex exponential

Compare with earlier result:

Fix w

$$S(e^{jw}, n) = e^{-jwn} \sum_{k} s(n-k)h(k)e^{jwk}$$

Comments on implementation

- Need to consider both required computation and nature of hardware to be used implementation

ex Winograd Fourier transform