Now suppose we move the window along as a function of time n:

n fixed vary w

$$\tilde{S}(e^{jw}, n) = \sum_k \underbrace{s(k)\,h(n-k)}_{\tilde{s}(\cancel{n,k})\,k,\,n)} e^{-jwk}$$

window is flipped around for mathematical convenience

With n fixed, this is just ordinary DTFT.

What does $\tilde{s}(k,n)$ look like?

Here k plays role of time
— n is a parameter giving us a family of signals

$\tilde{S}(e^{jw}, n)$ is just the DTFT of each member of the family

<u>Linear filtering interpretation</u>:

Can also write

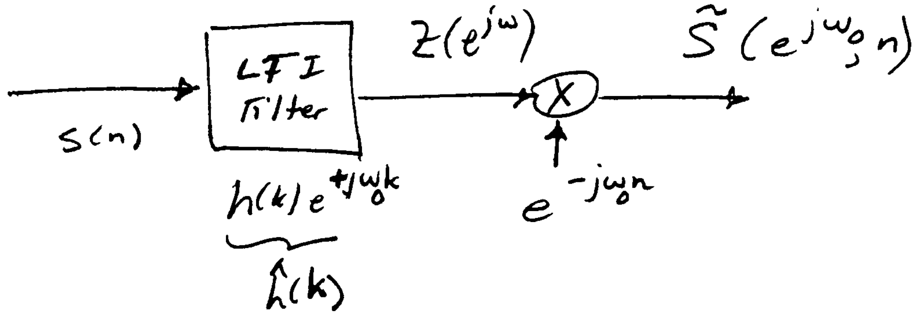
$$\tilde{S}(e^{jw}, n) = \sum_k s(n-k)\,h(k)\,e^{-jw(n-k)}$$

fix w vary n

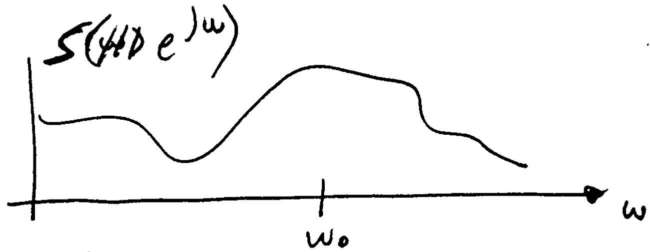
$$= e^{-jwn} \sum_k s(n-k)\,h(k)\,e^{+jwk}$$

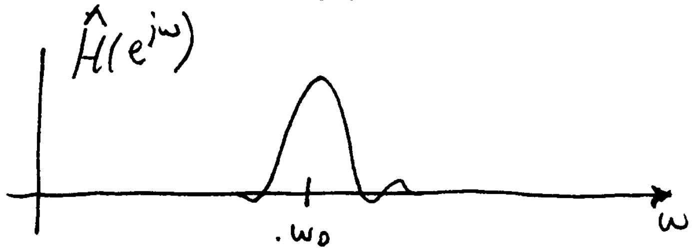
Let's look at these operations 1 by 1:

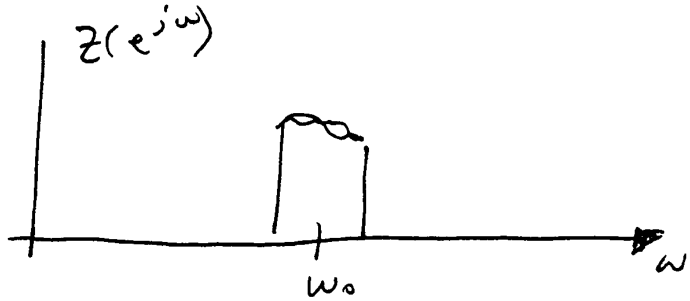
Let $w = w_0$ (fixed)



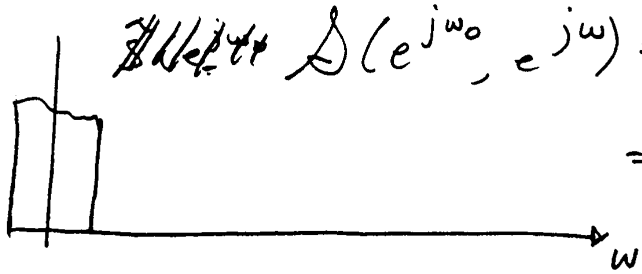
In the frequency domain:







$$\hat{S}(e^{jw_0}, e^{jw}) = \mathrm{DTFT}\{\tilde{S}(e^{jw_0}, n)\}$$
$$= \sum_n \tilde{S}(e^{jw_0}, n) e^{-jwn}$$



Examples

(1) $x[n] = \cos\left(\frac{\pi}{6} n\right)$

(2) $x[n] = \delta[n]$

(3) $x[n] = \begin{cases} \cos\left(\frac{\pi}{4} n\right), & n \cancel{\leq} < 0 \\ \cos\left(\frac{\pi}{12} n\right), & n \geq 0 \end{cases}$

(4) $x[n] = \begin{cases} \cos\left(\frac{\pi}{6} n\right), & 0 \leq n \bmod 20 \leq 9 \\ 0, & \text{else} \end{cases}$