This is all wonderful stuff; but suppose we don't know the underlying model that gave rise to our signal?

Estimating correlation

Assume X & Y are jtly w.s.s. processes, i.e.

$$E\{X[n]\} = \mu_x$$

$$E\{Y[n]\} = \mu_x$$

$$r_{xy}[m, m+n] = E\{X[m]\,Y[m+n]\}$$
$$= r_{xy}[n]$$

Suppose we form sum

$$Z[n] = \frac{1}{M}\sum_{m=0}^{M-1} X[m]\,Y[m+n]$$

a seq. of r.v.s

Consider

$$E\{Z[n]\} = \frac{1}{M}\sum_{m=0}^{M-1} E\{X[m]\,Y[m+n]\}$$

$$E\{z[n]\} = \frac{1}{M}\sum_{m=0}^{M-1} r_{XY}[n] = r_{XY}[n]$$

So we have an <u>unbiased</u> estimator of correlation.

Let's look a little more closely at what this computation involves:

Suppose we computed ~~$~~

$$\hat{r}_{XY}[n] = z[n] \quad \text{for } -N \le n \le N$$

$$= \frac{1}{M}\sum_{m=0}^{M-1} X[m]\, Y[m+n]$$

$n = -N$

$X[0] \quad X[1] \quad \cdots\cdots \quad X[M-1]$

$Y[-N] \quad Y[-N+1] \quad \cdots \quad Y[-N+M-1]$

$n = 0$

$X[0] \quad X[1] \quad \cdots \quad X[M-1]$

$Y[0] \quad Y[1] \quad \cdots \quad Y[M-1]$

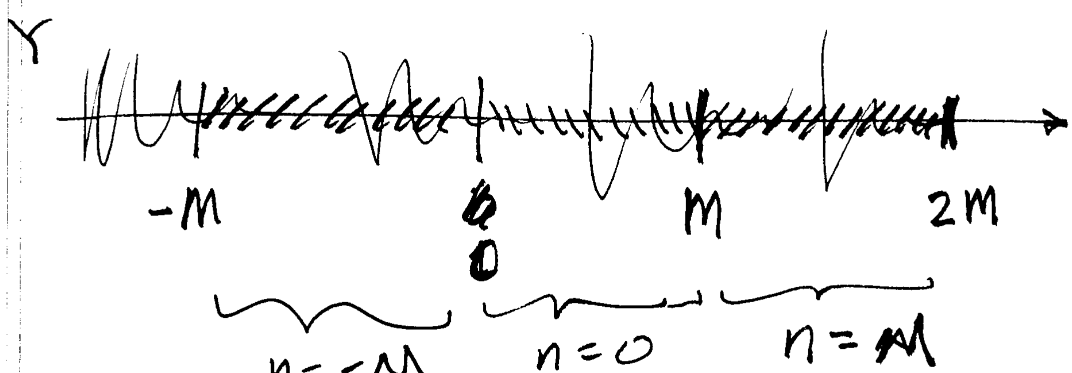
$n = N$

$X[0] \quad X[1] \quad \cdots \quad X[M-1]$

$Y[N] \quad Y[N+1] \quad \cdots \quad Y[N+M-1]$

In each case, we used same dataset for X, but data used for Y varied

Suppose $M = N$



Suppose instead we collect 2 sets of data

$X[0] \cdots X[M-1]$

$Y[0] \quad Y[M-1]$

and use all this data to compute estimate for every n

let $X_{tr}[n] = X[n]\{u[n] - u[n-M]\}$

$Y_{tr}[n] = Y[n]\{u[n] - u[n-M]\}$

Then let

$$V[n] = \sum_{m} X_{tr}[m]\, Y_{tr}[m+n]$$

$$= \sum_{m=0}^{M-1} X[m]\, Y[m+n] \Big\{ u[m+n] - u[m+n-M] \Big\}$$

Consider

$$E\{V[n]\} = \sum_{m=0}^{M-1} E\{X[m]\, Y[m+n]\} \Big\{ u[m+n] - u[m+n-M] \Big\}$$

$$= \sum_{m=0}^{M-1} r_{XY}[n] \Big\{ u[m+n] - u[m+n-M] \Big\}$$

$$(*) \qquad = r_{XY}[n] \sum_{m=0}^{M-1} \Big\{ u[m+n] - u[m+n-M] \Big\}$$

$$= r_{XY}[n]\, |M-n| \Big\{ u[n+(M-1)] - u[n-(M-1)] \Big\}$$

So an unbiased estimate of correlation is given by

$$\hat{r}_{xy}[n] = \left|\frac{1}{M-n}\right| \sum_{m=0}^{M-1} X[m]\, Y_{tr}[m+n]$$

$$-(M-1) \le n \le M-1$$

Note that as $|n|$ increases estimate becomes increasing noisy.

check on (*)

For $n = M-1$, have

$$\sum_{m=0}^{M-1} \{ u[m+(M-1)] - u[\underbrace{m+(M-1)-M}_{u[m-1]}] \}$$

on at $m = -(M-1)$

off at $m = 1$

so get only 1 term

For $n = -(M-1)$, have

$$\sum_{m=0}^{M-1} \{ u[m-(M-1)] - u[m-(M-1)-M] \}$$

on at $m = M-1$

off at $m = 2M-1$

so again get only 1 term in sum