Sequences of random variables

$X_1, X_2, X_3, \ldots, X_N$

Behavior completely characterized by

$$P\{x_1^l \le X_1 \le x_1^u,\ x_2^l \le X_2 \le x_2^u, \ldots, x_N^l \le X_N \le x_N^u\}$$

$$= \int_{x_1^l}^{x_1^u} \int_{x_2^l}^{x_2^u} \cdots \int_{x_N^l}^{x_N^u} f_{X_1 X_2 \cdots X_N}(x_1, x_2, \ldots, x_N)\, dx_1\, dx_2 \cdots dx_N \quad (1)$$

It is apparent that this can get complicated very quickly.

In practice, consider only 2 cases:

1. $X_1, X_2, \ldots, X_N$ mutually independent

$$\Rightarrow f_{X_1, X_2, \ldots, X_N}(x_1, x_2, \ldots, x_N) = \prod_{i=1}^{N} f_{X_i}(x_i)$$

In ~~this~~ (1) above factors into a product of integrals

2. $X_1, X_2, \ldots, X_N$ jointly Gaussian

(discuss later)

- Example

$X_1 \cdots X_n$ i.i.d stats. ~~mean~~ μ_X σ_X^2

$N_1 - \cdot N_n$ i.i.d. zero mean

$Y_i = aX_i + N_i$

$E\{Y_i\} = a\,E\{X_i\} + E\{N_i\}$

$\overline{Y} = a\overline{X}$

- $\overline{Y^2} = E\{(aX_i + N_i)^2\}$

$= a^2\overline{X^2}\,\cancel{} + \underbrace{\overline{N^2}}_{\sigma_N^2}$

$\sigma_Y^2 = \overline{Y^2} - (\overline{Y})^2$

$= a^2\overline{X^2} + \sigma_N^2 - a^2(\overline{X})^2$

$= a^2\sigma_X^2 + \sigma_N^2$

- $\overline{XY} = E\{X_i(aX_i + N_i)\}$

$= a\overline{X^2}$

$$\sigma_{xy}^2 = \frac{a\overline{x^2} - a(\bar{x})^2}{\sigma_x (a^2\sigma_x^2 + \sigma_N^2)^{1/2}}$$

$$= a \frac{\sigma_x}{\left(\sqrt{a^2\sigma_x^2 + \sigma_N^2}\right)^{1/2}}$$

$$= \cancel{\frac{1}{\sqrt{1 + \frac{1}{a^2 SNR}}}}$$

$$= \frac{1}{\sqrt{1 + \sigma_N^2 / (a^2\sigma_x^2)}}$$