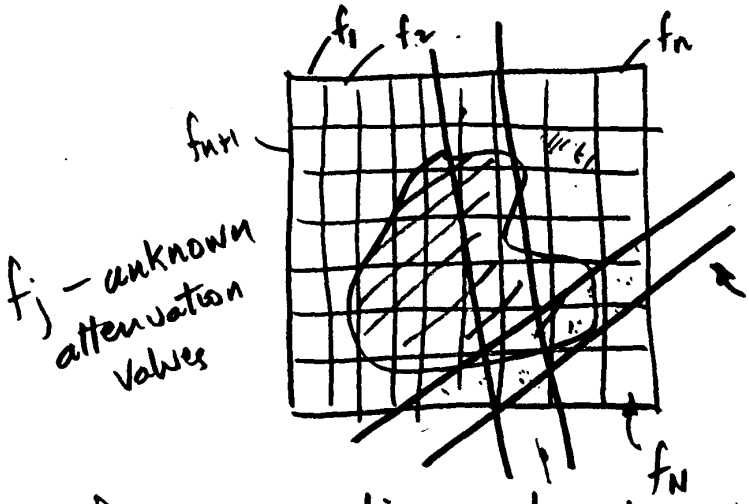
Algebraic Reconstruction Technique (ART)



Discretize line integral

measured ray sums $p_i = \sum_{j=1}^{N} w_{ij} f_j$ $\quad i = 1, \ldots, M$ total of M line integrals

weights depend only on geometry — image independent

Have a set of linear equations

M eq.s & N unknowns

$M = N$ invert W to solve for f_j's

$M > N$ overdetermined (least-squares)

$M < N$ underdetermined (sol. not unique)

Typically $N \sim (512)^2$

Iterative Solution

let k be iteration index

let $f_j^k \quad j = 1, \ldots, N$ be estimated attenuation after k-th iteration

Let $p_i^k = \sum_{j=1}^{N} w_{ij} f_j^k$ ← known

be the i-th ray sum based on estimate of attenuation

Compare to measured ray sums:

$$e_i^k = p_i^k - p_i$$

Correct the pixels to yield no error in i-th ray sum:

$$f_j^{k+1} = f_j^k - \frac{w_{ij} e_i^k}{\sum_{j=1}^{N} w_{ij}^2}$$

Can show that (HW problem)

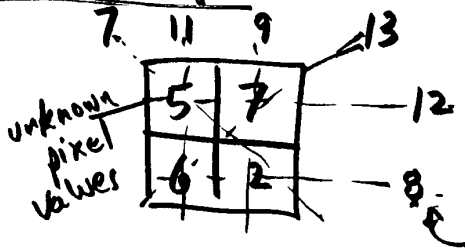
$$\sum_{j=1}^{N} w_{ij} f_j^{k+1} = p_i$$

Repeat process for each ray $i = 1, \ldots, M$
Then repeat as necessary until procedure converges.

example of a "projection" onto convex sets (POCS)

Simple Example

assume $w_{ij} \equiv 0$ or 1



Initial guess