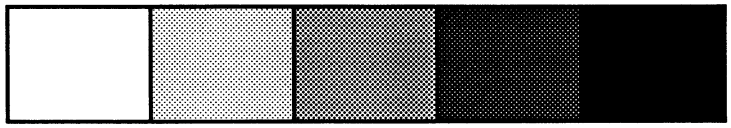
2.3.5 HALFTONING

- Used for representation of continuous-tone with devices that are bi-level, or which can generate more than two output levels but not a sufficient number of levels to prevent the appearance of quantization artifacts.

- All halftoning techniques rely on a local spatial average over binary textures by the human viewer to create the impression of continuous-tone.

- Detail is rendered by locally modulating these textures.

Units for Gray-Value (Ideal)

Texture 

Digital Value	255	191	127	63	0
Absorptance	0.0	0.25	0.5	0.75	1.0
Reflectance/ Transmittance	1.0	0.75	0.5	0.25	0.0

Notation

$0 \le f[m,n] \le 1$, digital, continuous-tone original image

$g[m,n] = 0,\ 1,$ digital halftone image

$g(x,y)$ – displayed/printed halftone image

Model for Printed/Displayed Images

$$g(x,y) = \sum_{m}\sum_{n} g[m,n]\,p_S(x - mR,\ y - nR)$$

- device-addressable points lie on a square lattice with interval $R \times R$
- $p_S(x,y)$ - printed/displayed spot profile
- if there is spot overlap, it is assumed to be additive.

Halftoning Techniques

1. Binarization with a constant threshold

2. Pattern printing

3. Screening

4. Error diffusion

Binarization with a Constant Threshold

$$g[m,n] = \begin{cases} 1, & f[m,n] \geq 0.5 \\ 0, & \text{else} \end{cases}$$

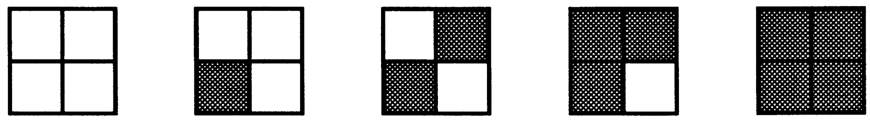
- minimizes mean-squared error

$$E = \sum_m \sum_n |f[m,n] - g[m,n]|^2$$

- does not yield acceptable quality

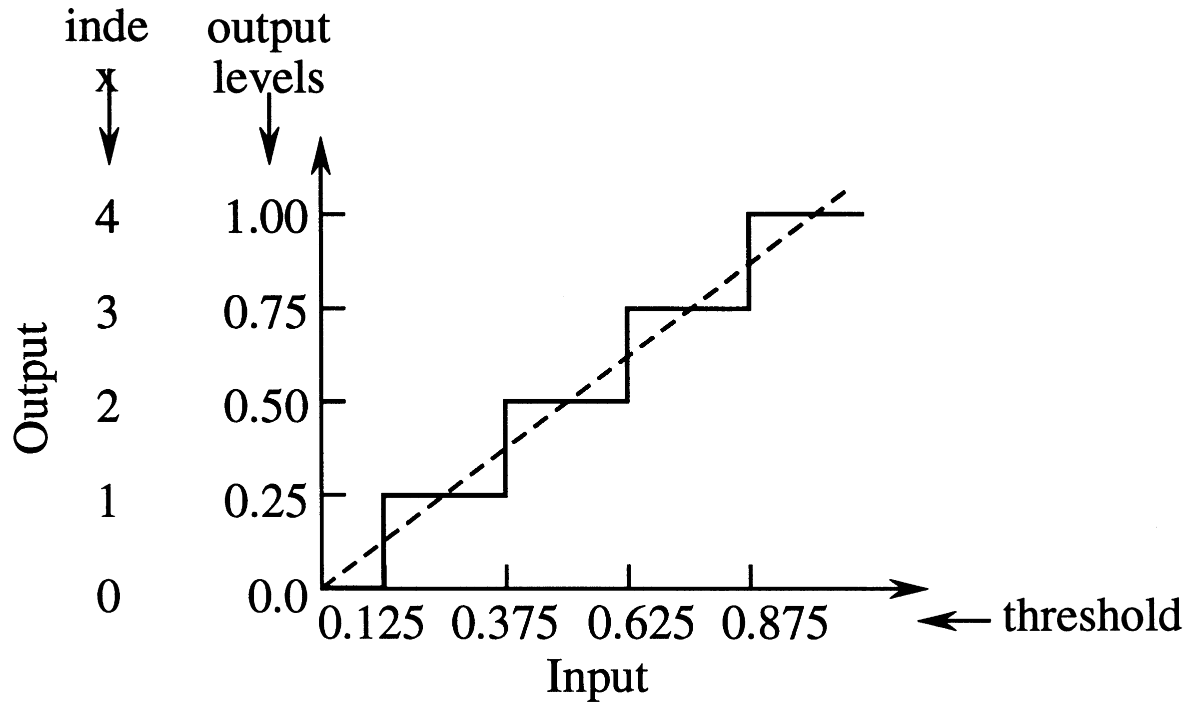
PATTERN PRINTING

- pattern library $p[m,n;\ell]$

Binary Pattern					
Index ℓ	0	1	2	3	4
Average Absorptance	0.00	0.25	0.50	0.75	1.00

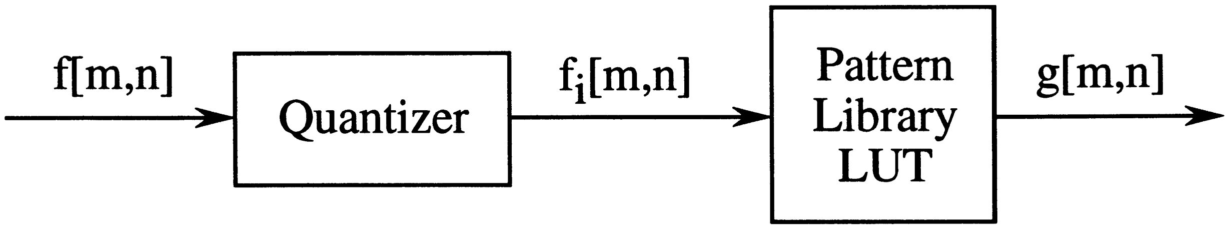
- $M \times N$ patterns yield $MN + 1$ output quantization levels (Here $M = N = 2$).

- quantizer design



- Mapping to index image

$$f_i[m,n] = \ell : [\ell - 1/2]/MN < f[m,n] \leq [\ell + 1/2]/MN$$

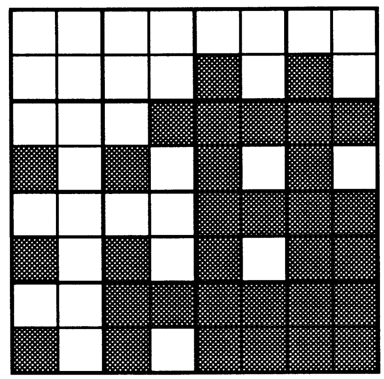


0.1	0.1	0.3	0.3
0.2	0.4	0.7	0.7
0.2	0.3	0.7	0.9
0.3	0.7	0.9	0.9

$f[m,n]$

0	0	1	1
1	2	3	3
1	1	3	4
1	3	4	4

$f_i[m,n]$



$g[m,n]$

- Halftone image is larger than continuous-tone original by factor $M \times N$.

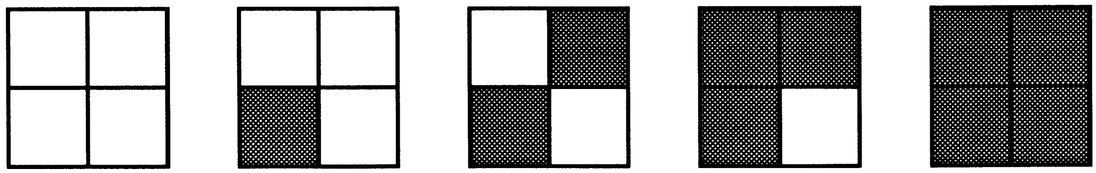
- If device resolution is sufficiently high, pattern printing will yield acceptable results.

- At lower resolution, images appear blocky and lack detail.

- There is a tradeoff between detail resolution and number of quantization levels.

Alternate Representations for Pattern Library

- Dot profile function $p[m,n;\ell]$



ℓ 0 1 2 3 4

- Index matrix

3	2
1	4

- Entries indicate order in which dots are added to binary structure
- Stacking constraint must be satisfied

Stacking Constraint

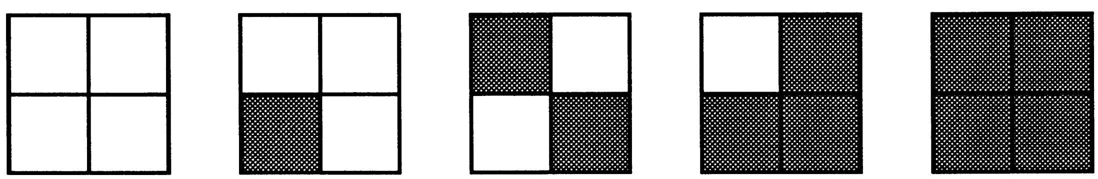
For any $0 \le \ell \le MN$,

$$p[m,n;\ell] = 1 \;\Rightarrow\; p[m,n;k] = 1 \quad \forall k \ge \ell$$

or

$$p[m,n;\ell] = 0 \;\Rightarrow\; p[m,n;\ell] = 0 \quad \forall k \le \ell$$

- A dot profile that does not satisfy this constraint:



0 1 2 3 4

Alternate Representations for Pattern Library (cont.)

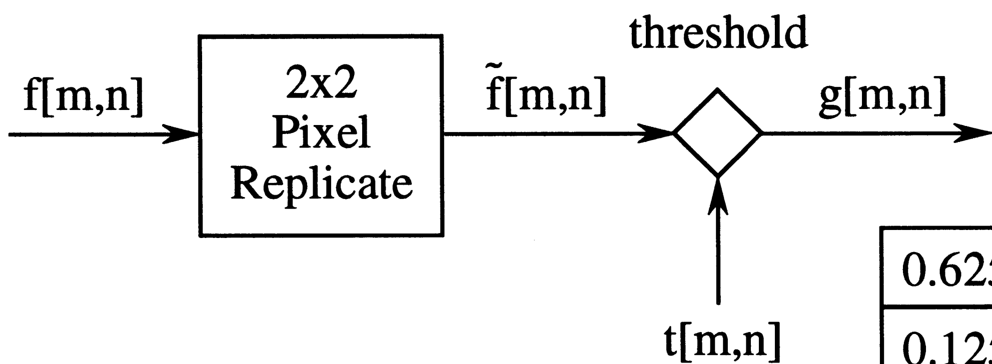
- Index matrix i[m,n]

3	2
1	4

- Threshold Matrix t[m,n]
  t[m,n]=(i[m,n] - 0.5)/MN

0.625	0.375
0.125	0.875

Alternate Implementation for Pattern Printing



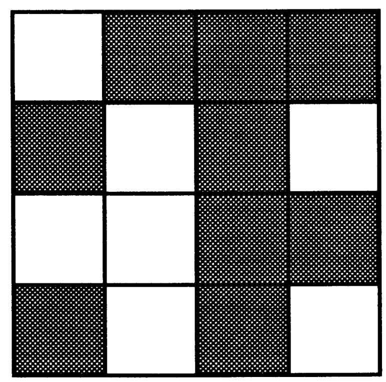
0.625	0.375
0.125	0.875

0.4	0.7
0.3	0.7

f[m,n]

0.4	0.4	0.7	0.7
0.4	0.4	0.7	0.7
0.3	0.3	0.7	0.7
0.3	0.3	0.7	0.7

$\tilde{f}[m,n]$

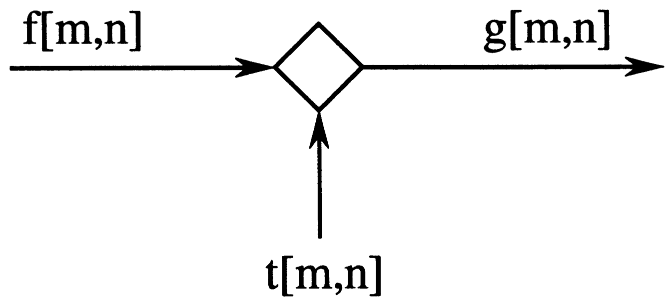


g[m,n]

- threshold signal is doubly periodic

$$t[m,n] = t[m + kM,\ n + \ell N]$$

SCREENING



$$g[m,n] = \begin{cases} 1, & f[m,n] \geq t[m,n] \\ 0, & \text{else} \end{cases}$$

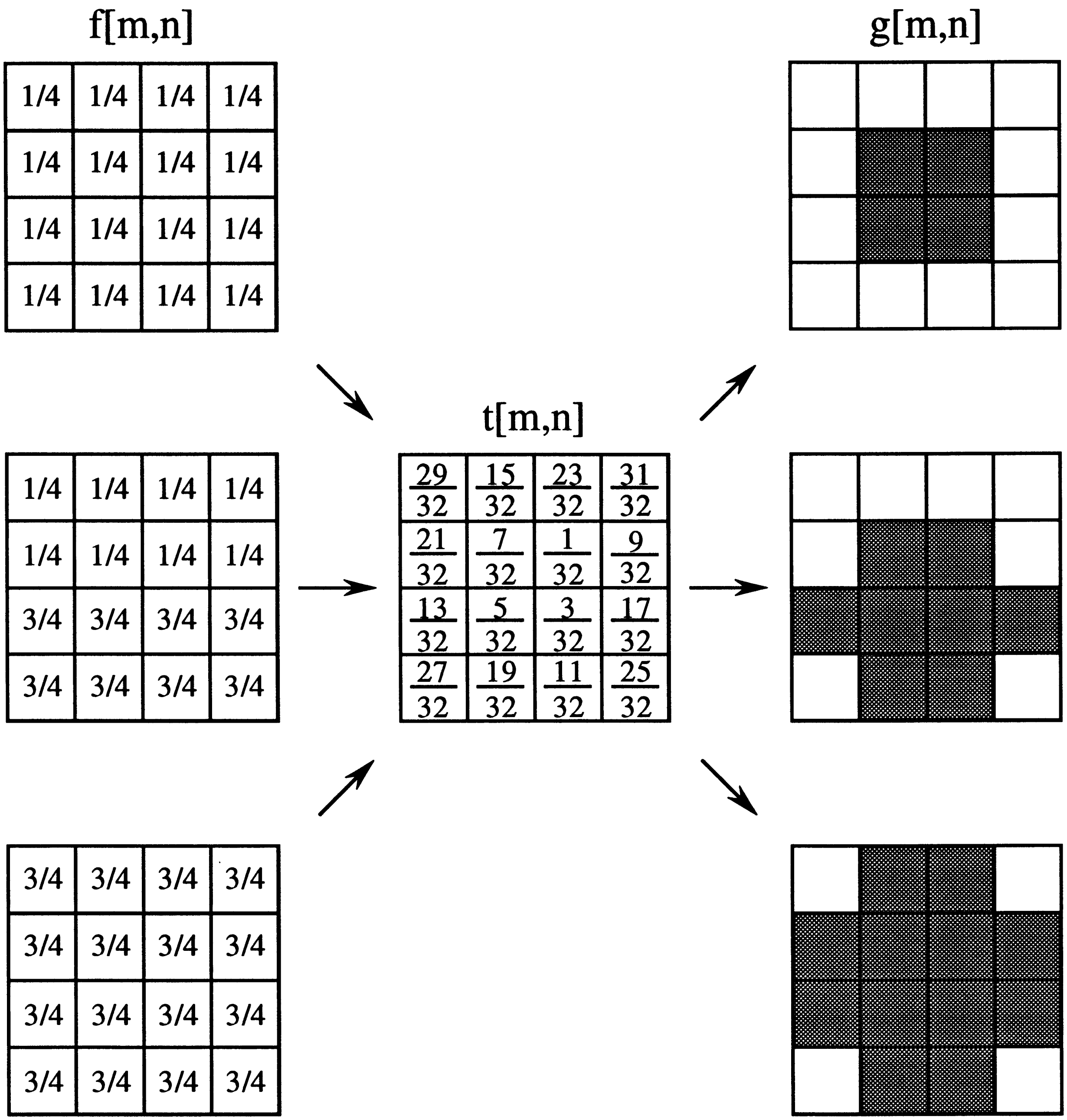
- Halftone image is same size as continuous-tone original image.

- Technique is equivalent to photographic contact screening process traditionally used in graphic arts and printing.

- Dot profile function must satisfy stacking constraint.

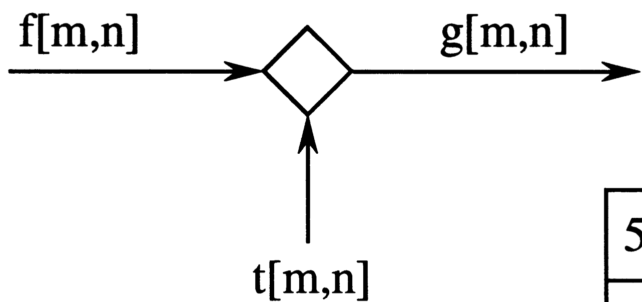
- Screening achieves better detail rendition than pattern printing via partial dotting property.

Partial Dotting



Different Representations for Screening

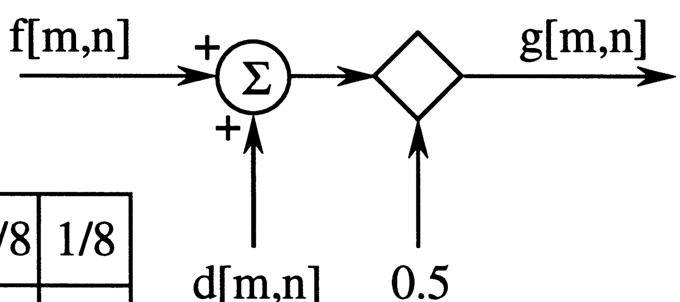
1. Spatially varying threshold



$$g[m,n] = \begin{cases} 1, & f[m,n] \geq t[m,n] \\ 0, & \text{else} \end{cases}$$

5/8	3/8
1/8	7/8

2. Addition of dither signal



$$g[m,n] = \begin{cases} 1, & f[m,n] + d[m,n] \geq 0.5 \\ 0, & \text{else} \end{cases}$$

-1/8	1/8
3/8	-3/8

3. Point-to-Point Nonlinear Mapping Via Dot Profile Function

Input Gray Level b	$0 \le b < 1/8$	$1/8 \le b < 3/8$	$3/8 \le b < 5/8$	$5/8 \le b < 7/8$	$7/8 \le b < 1$
Binary Pattern p[m,n;b]	$\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$	$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$	$\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$	$\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

- $p[m + kM,\ n + \ell N;\ b] = p[m,n;b]$
- $g[m,n] = p[m,n;\ f[m,n]]$

Choice of Threshold Matrix (Screen Function)

- Size of matrix (M and N) determines period of screen and number of quantization levels.

- Thresholds are chosen to yield correct tone reproduction (minimum quantization error).

- Spatial arrangement of the thresholds determines characteristics of the texture that results.

Recall Dual Representation

Index Matrix	
3	2
1	4

i[m,n]

Threshold Matrix	
5/8	3/8
1/8	7/8

t[m,n]=(i[m,n] - 0.5)/MN

Clustered Dot Screen

63	58	49	37	38	50	59	64
57	48	36	22	23	39	51	60
47	35	21	11	12	24	40	52
34	20	10	4	1	5	13	25
33	19	9	3	2	6	14	26
46	32	18	8	7	15	27	41
56	45	31	17	16	28	42	53
62	55	44	30	29	43	54	61

i[m,n]

- Consecutive thresholds are located in close spatial proximity.

Properties of Clustered Dot Screen

1. Relatively visible texture

2. Relatively poor detail rendition

3. Uniform texture across entire grayscale

4. Robust performance with non-ideal output devices

   - non-additive spot overlap
   - spot-to-spot variability
   - noise

Dispersed Dot Screen

Bayer's Optimum Index Matrix (1973)

Recursive Definition (Judice, Jarvis, Ninke, 1974)

1. Let $i'[m,n]$ be any $M \times N$ index matrix
2. Define a new $2M \times 2N$ index matrix $i[m,n]$ as

$$\left[\begin{array}{c|c} 4(i'[m,n]-1)+3 & 4(i'[m,n]-1)+2 \\ \hline 4(i'[m,n]-1)+1 & 4(i'[m,n]-1)+4 \end{array}\right]$$

$i[m,n]$

3. Recursively generate $2^K \times 2^K$ matrix starting with 1×1 index matrix [1].

Example

3	2
1	4

12	8	10	6
4	16	2	14
9	5	11	7
1	13	3	15

48	32	38	24	46	30	36	22
16	64	8	56	14	62	6	54
36	20	44	28	34	18	42	26
4	52	12	58	2	50	10	56
45	29	35	21	47	31	37	23
13	61	5	53	15	63	7	55
33	17	41	25	35	19	43	27
1	49	9	55	3	51	11	57

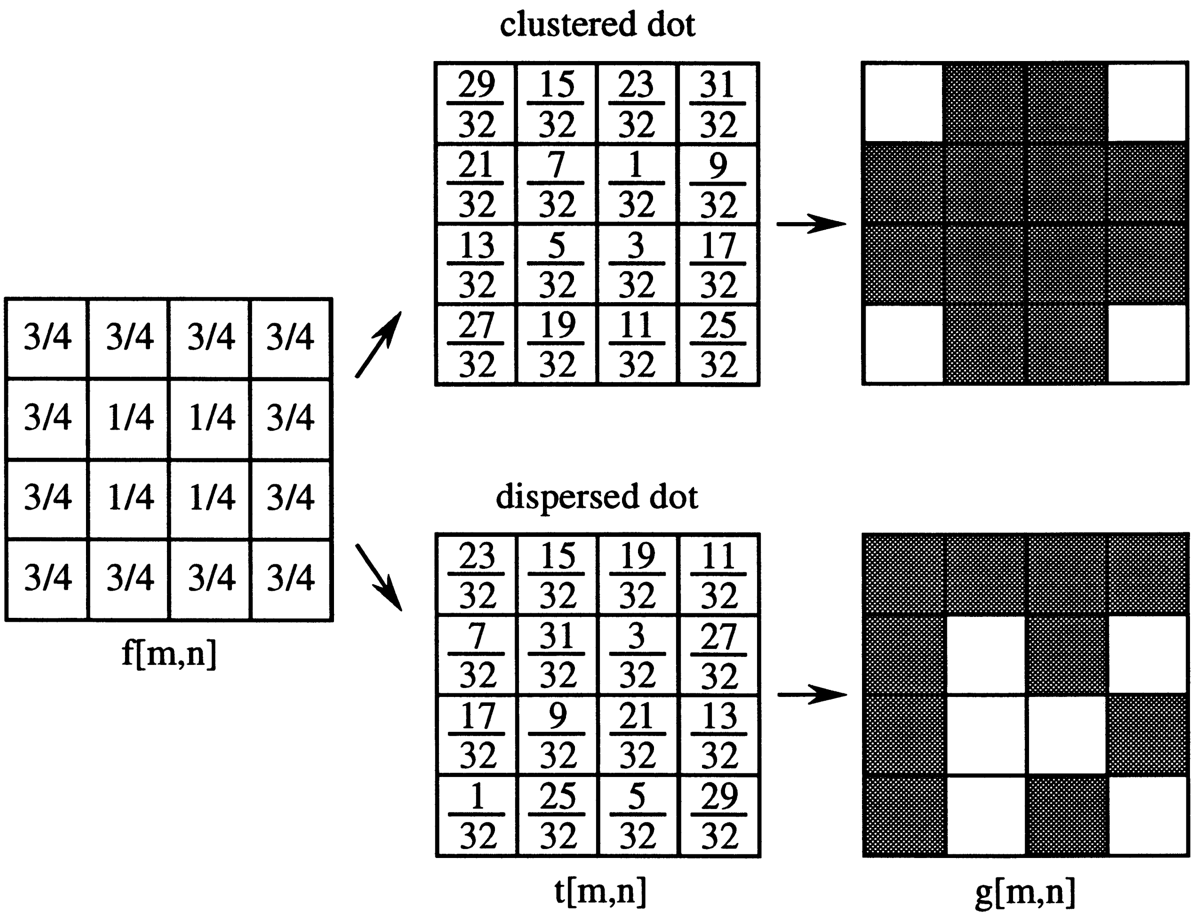
- Consecutive threshold are located far apart spatially.

Recursive Definition for Threshold Matrix

$$t[m,n] = \left[\begin{array}{c|c} t'[m,n] + \dfrac{0.5}{4MN} & t'[m,n] - \dfrac{0.5}{4MN} \\ \hline t'[m,n] - \dfrac{1.5}{4MN} & t'[m,n] + \dfrac{1.5}{4MN} \end{array}\right]$$

- Yields finer amplitude quantization over larger $(2M \times 2N)$ area.
- Retains good detail rendition within smaller $M \times N$ regions.

Example illustrating improved detail rendition with a dispersed dot screen



Properties of Dispersed Dot Screen

1. Within any region containing K dots, the K thresholds should be distributed as uniformly as possible between 0 and 1.

2. Textures used to represent individual gray levels have low visibility.

3. Improved detail rendition.

4. Transition between textures corresponding to different gray levels may be more visible.

5. Poor performance with non-ideal output devices

FOURIER ANALYSIS

1. Screening

- Continuous-tone, continuous-parameter original image

$$f(x,y) \overset{\text{CSFT}}{\leftrightarrow} F(u,v)$$

$$f[m,n] = f(mR,\ nR)$$

- Halftone image

$$g(x,y) \overset{\text{CSFT}}{\leftrightarrow} G(u,v)$$

$$g(x,y) = \sum_m \sum_n g[m,n] p_S(x - mR,\ y - nR)$$

$$p_S(x,y) \overset{\text{CSFT}}{\leftrightarrow} P_S(u,v)$$

Definition of Transforms

- Continuous-space Fourier transform (CSFT)

$$F(u,v) = \iint f(x,y) e^{-j2\pi(ux+vy)} dxdy$$

- Discrete Fourier transform (DFT)

$$P[k,\ell;b] = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} p[m,n;b] e^{-j2\pi(\frac{mk}{M} + \frac{n\ell}{N})}$$

- Dot profile function ($M \times N$ period)

$$p[m,n;b] \overset{\text{DFT}}{\leftrightarrow} P[k,\ell;b]$$

$$g[m,n] = p[m,n;\, f[m,n]]$$

- Halftone cell - $X \times Y$ $\quad X = MR, \quad Y = NR$

Spectrum of Halftone Image

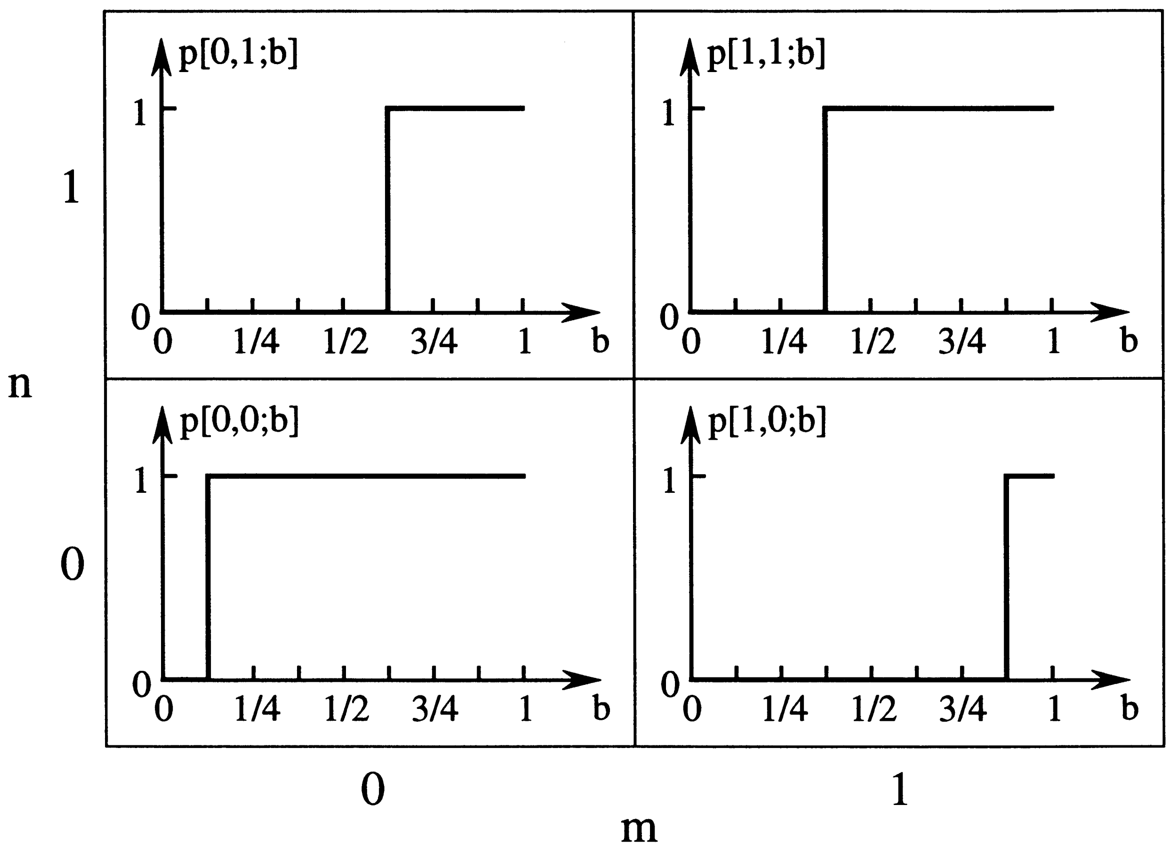
$$G(u,v) = P_s(u,v) \sum_m \sum_n F_{mn}(u-m/X,\ v-n/Y)$$

$$F_{mn}(u,v) = \mathrm{CSFT}\{f_{mn}(x,y)\}$$

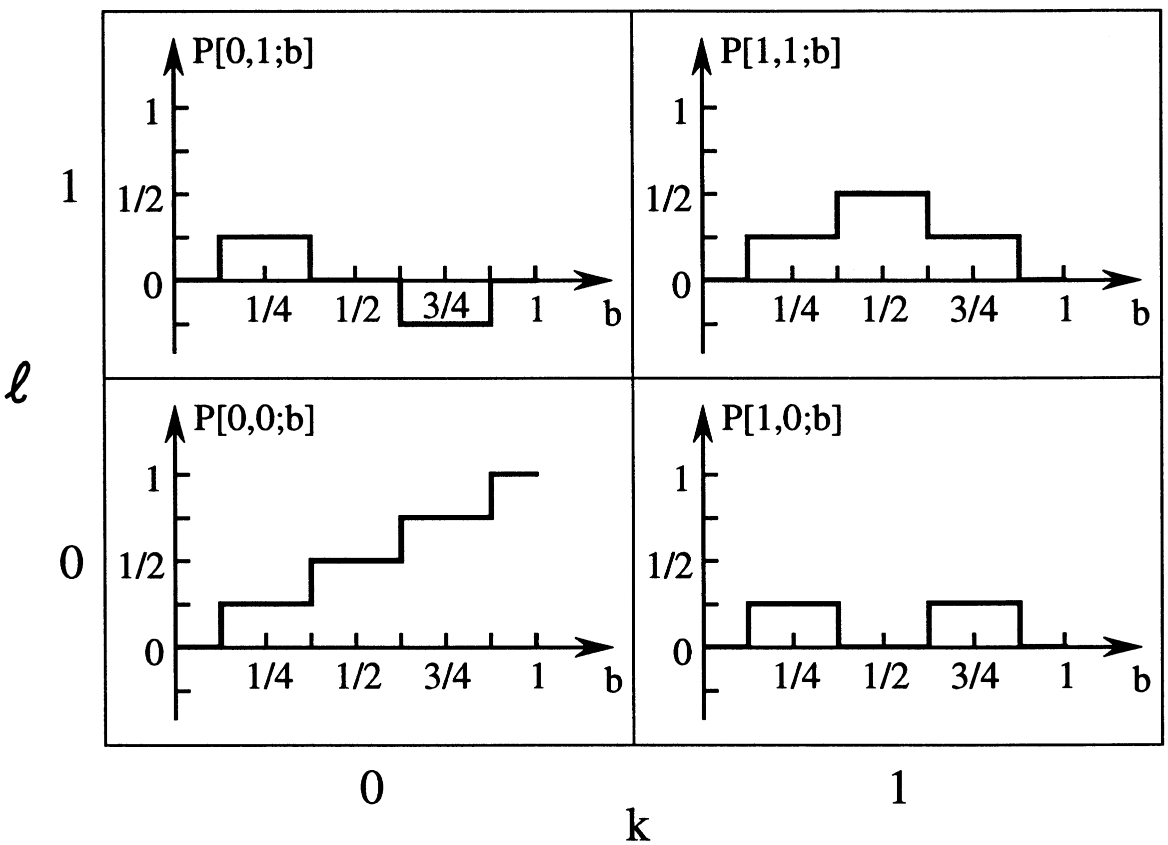
$$f_{mn}(x,y) = P[m,n;\ f(x,y)]$$

Relation Between Dot Profile and Spectral Nonlinearities

Input Gray Level b	$b \in$	[0, 1/8)	[1/8, 3/8)	[3/8, 5/8)	[5/8, 7/8)	[7/8, 1)
Binary Pattern p[m,n;b]	n = 1	0 0	0 0	0 1	1 1	1 1
	n = 0	0 0	1 0	1 0	1 0	1 1
	m	0 1				


p[0,1;b]
p[1,1;b]
p[0,0;b]
p[1,0;b]
1
0
0
1/4
1/2
3/4
1
b
n
m

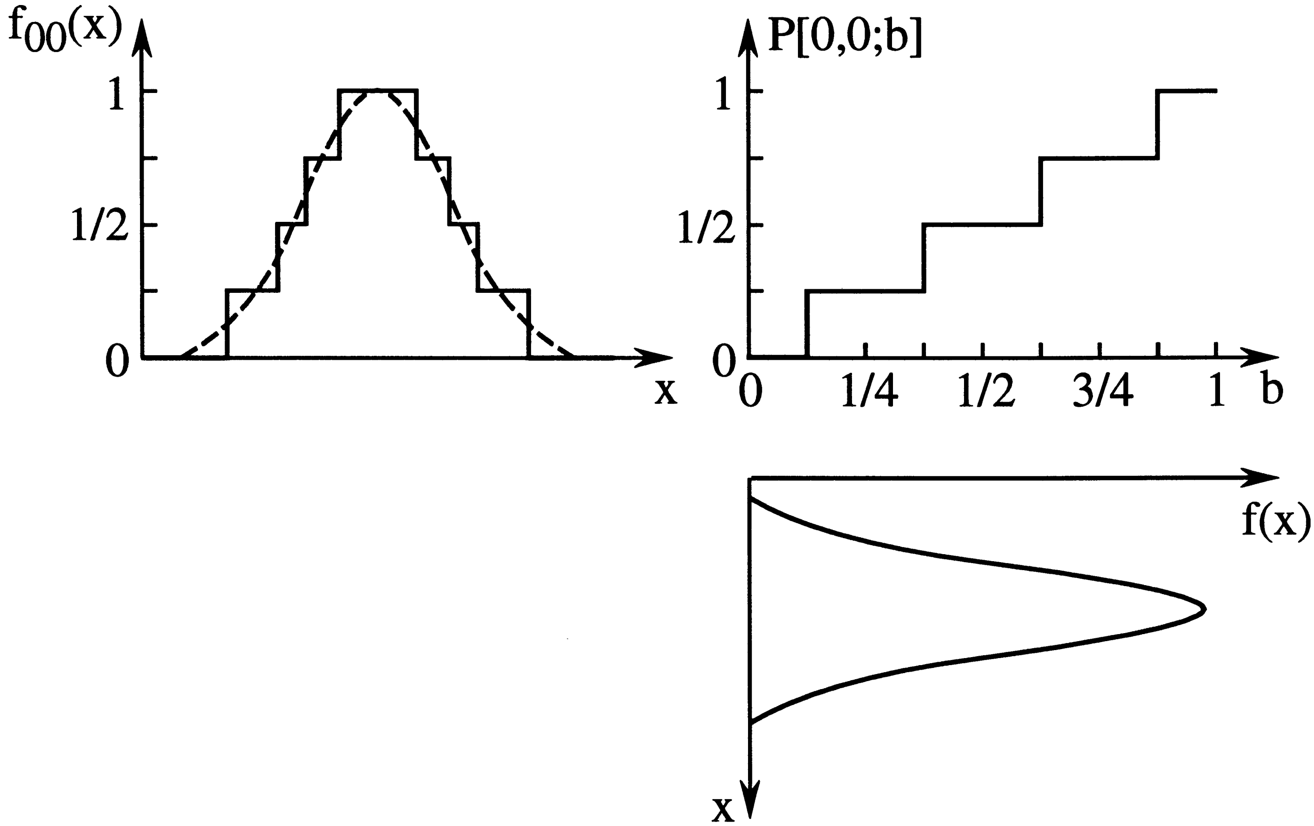
Input Gray Level b	ℓ	[0, 1/8)		[1/8, 3/8)		[3/8, 5/8)		[5/8, 7/8)		[7/8, 1)	
DFT $P[k,\ell;b]$	1	0	0	1/4	1/4	0	1/2	1/4	1/4	0	0
	0	0	0	1/4	1/4	1/2	0	3/4	1/4	1	0
	k	0	1								

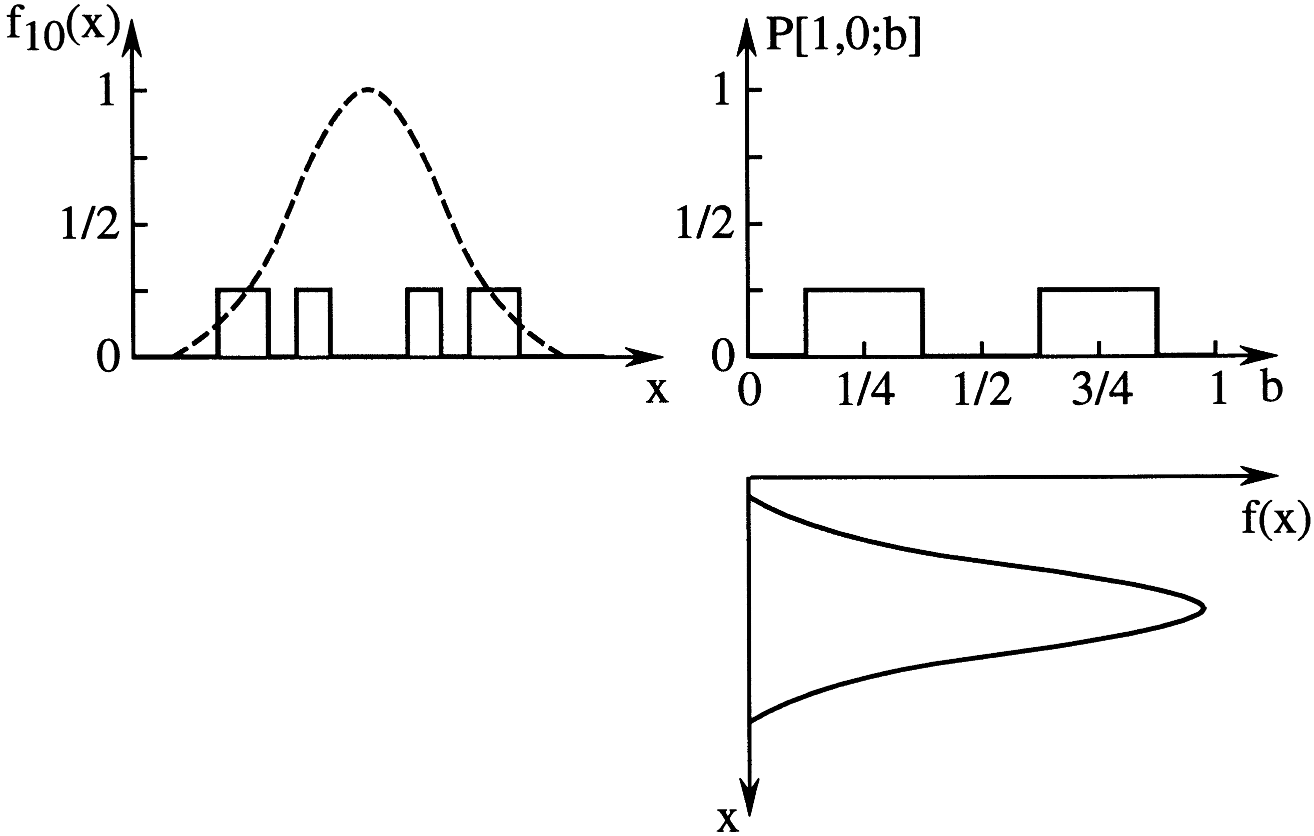

P[0,1;b]
P[1,1;b]
P[0,0;b]
P[1,0;b]
1
1/2
0
1/4
1/2
3/4
1
b
ℓ
k

Nonlinearly Transformed Images

$$f_{mn}(x,y) = P[m,n;f(x,y)]$$

1-D Example





f10(x)
1
1/2
0
x
P[1,0;b]
1
1/2
0
0
1/4
1/2
3/4
1
b
f(x)
x

2. Pattern Printing

- Continuous-tone, continuous-parameter original image

$$f(x,y) \overset{\text{CSFT}}{\leftrightarrow} F(u,v)$$

- Halftone cell - $X \times Y$, $X = MR$, $Y = NR$

- Sample-and-hold image

$$\tilde{f}(x,y) = \text{rect}\left(\frac{x}{X}, \frac{y}{X}\right) ** \text{comb}_{XY}[f(x,y)]$$

$$\tilde{F}(u,v) = \text{sinc}(Xu, Yv)\,\text{rep}_{\frac{1}{X}\frac{1}{Y}}[F(u,v)]$$

- In analysis of screening, replace $f(x,y)$ by $\tilde{f}(x,y)$ and $F(u,v)$ by $\tilde{F}(u,v)$.

Other Screen Functions

- Optimized Threshold Matrices (Allebach and Stradling, 1979)
- Angled Screens (Holladay, 1980)
- Macroscreens

ERROR DIFFUSION

Definition of terms

- Continuous-tone, discrete parameter, original image - $f[m,n]$
- Modified continuous-tone image - $\tilde{f}[m,n]$
- Diffusion weights - $w[k,\ell]$

$$w[k,\ell] \geq 0, \quad \sum_k \sum_\ell w[k,\ell] = 1$$

- Halftone image - $g[m,n]$

Description of algorithm

- Start with $\tilde{f}[m,n] \equiv f[m,n]$
- Scan pixels in image in a predetermined order, and carry out following computations

threshold

$$g[m,n] = \begin{cases} 1\,, & \tilde{f}[m,n] \geq 0.5 \\ 0\,, & \text{else} \end{cases}$$

compute error

$$e[m,n] = g[m,n] - \tilde{f}[m,n]$$

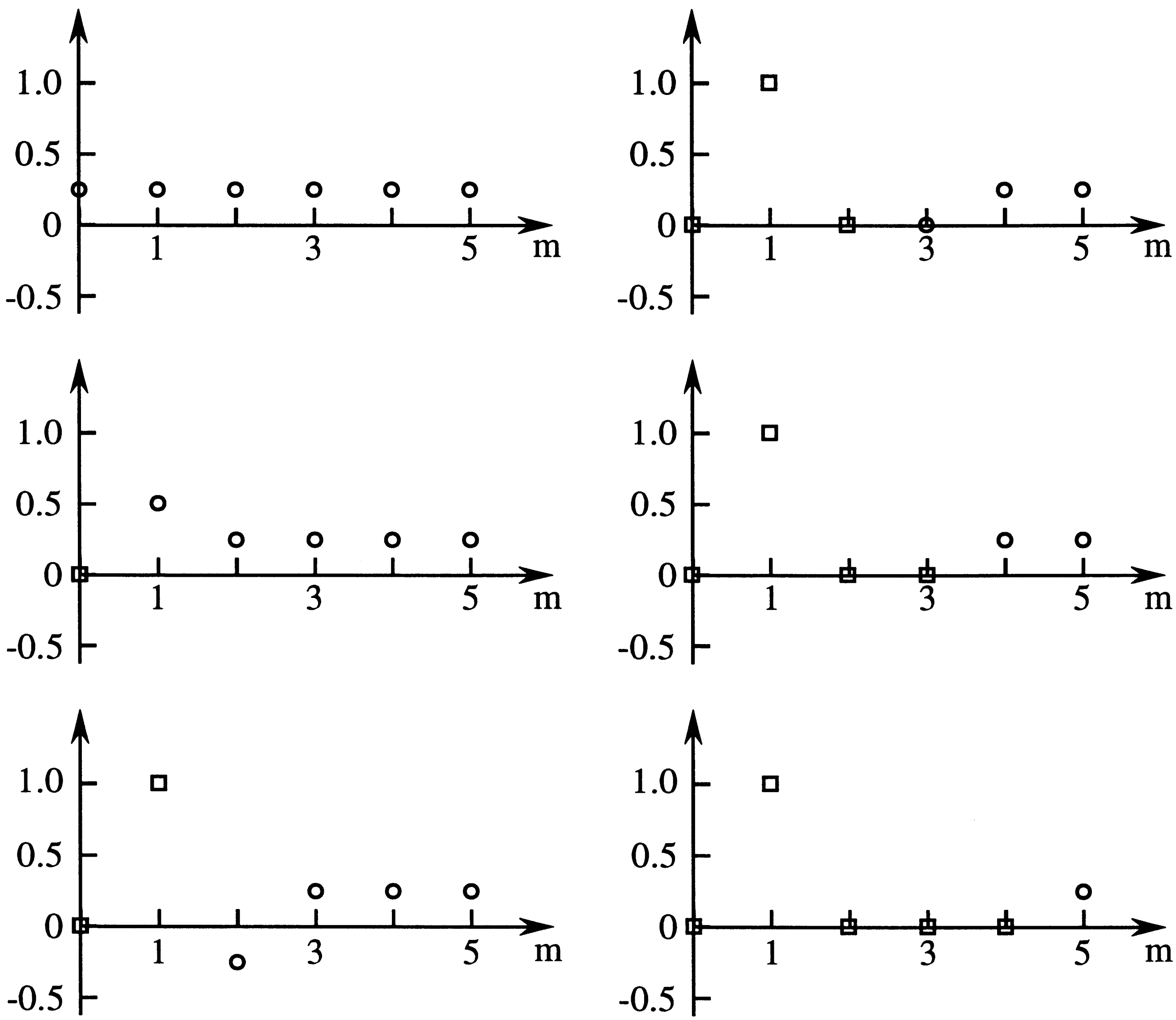
diffuse error

$$\tilde{f}[m+k,\ n+\ell] = \tilde{f}[m+k,\ n+\ell] - w[k,\ell]\ e[m,n]$$

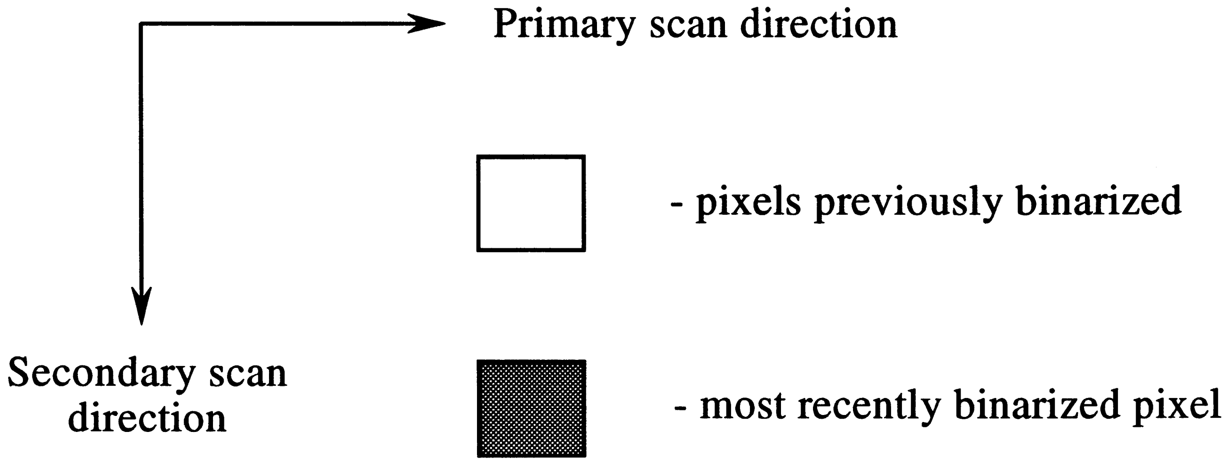
$$(m+k,\ n+\ell) \in \{\text{pixels not yet binarized}\}$$

1-D Example

$f[m] \equiv 0.25 \qquad \tilde{f}[m] - \bigcirc \qquad g[m] - \square$



2-D Error Diffusion Weighting Filters



		7/16
3/16	5/16	1/16

Floyd, and Steinberg (1976)

			7/48	5/48
3/48	5/48	7/48	5/48	3/48
1/48	3/48	5/48	3/48	1/48

Jarvis, Judice, and Ninke (1976)

Characteristics of Error Diffusion

- At each step, error diffusion preserves local average over part of image that has been binarized and part that is yet to be binarized.
- No fixed number of quantization levels.
- Requires more computation than screening.
- Excellent detail rendition (sharpens image).
- Generally good texture with some exceptions:
  - texture contouring
  - worm-like patterns
  - texture used to render a given gray level is context-dependent

FOURIER ANALYSIS (Knox, 1991)

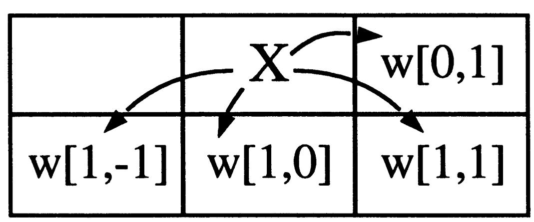
Two Views of Error Diffusion

1. Diffuse error immediately after binarizing pixel to all pixels in neighborhood

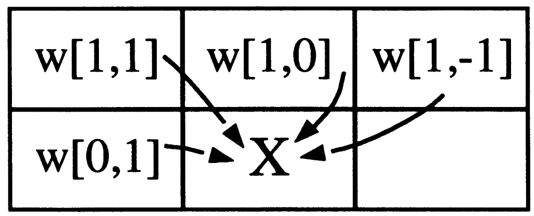
$$g[m,n] = \begin{cases} 1, & \tilde{f}[m,n] \geq 0.5 \\ 0, & \text{else} \end{cases}$$

$$e[m,n] = g[m,n] - \tilde{f}[m,n]$$

$$\tilde{f}[m+k,\ n+\ell] = \tilde{f}[m+k,\ n+\ell] - w[k,\ell]e[m,n]$$



2. Diffuse error from all neighboring pixels to pixel to be binarized, just prior to binarization



$$\tilde{f}[m,n] = f[m,n] - \sum_{k}\sum_{\ell} w[k,\ell]\,e[m-k,\, n-\ell] \qquad (1)$$

$$g[m,n] = \begin{cases} 1, & \tilde{f}[m,n] \geq 0.5 \\ 0, & \text{else} \end{cases} \qquad (2)$$

$$e[m,n] = g[m,n] - \tilde{f}[m,n] \qquad (3)$$

Recursive Expression for the Error Image

Combine Eqs. (1) and (2)

$$e[m,n] = g[m,n] - f[m,n] + \sum_k \sum_\ell w[k,\ell]e[m-k,\ n-\ell]$$

Discrete-Space Fourier Transform (DSFT)

$$E(\mu,\nu) = \sum_m \sum_n e[m,n]e^{-j(m\mu+n\nu)}$$

$$E(\mu,\nu) = G(\mu,\nu) - F(\mu,\nu) + W(\mu,\nu)E(\mu,\nu)$$

- We would like an expression for $G(\mu,\nu)$ in terms of $F(\mu,\nu)$

- Instead, we have

$$G(\mu,\nu) = F(\mu,\nu) + \overline{W}(\mu,\nu)E(\mu,\nu)$$

- High-pass filter

$$\overline{W}(\mu,\nu) = 1 - W(\mu,\nu)$$

- Error spectrum is not known

$$E(\mu,\nu) = G(\mu,\nu) - \tilde{F}(\mu,\nu)$$

Error Model

$$E(\mu,\nu) = cF(\mu,\nu) + R(\mu,\nu)$$

- Original image component $cF(\mu,\nu)$; constant c depends on weighting and input image

weighting	**c**
1-D	0.0
Floyd and Steinberg	0.55
Jarvis, Judice, and Ninke	0.80

- Residual $R(\mu,\nu)$ - may still be image dependent

Edge-Enhancing Property of Error Diffusion

- Combine

$$G(\mu,\nu) = G(\mu,\nu) + \overline{W}(\mu,\nu)E(\mu,\nu) \quad \text{and}$$

$$E(\mu,\nu) = cF(\mu,\nu) + R(\mu,\nu)$$

$$G(\mu,\nu) = [1 + c\overline{W}(\mu,\nu)]F(\mu,\nu) + \overline{W}(\mu,\nu)R(\mu,\nu)$$

- Edge-Enhancing Filter $1 + c\overline{W}(\mu,\nu)$

- Blue Noise $\overline{W}(\mu,\nu)R(\mu,\nu)$