2.3.2 GENERAL MODEL FOR DISPLAY AND PRINTING PROCESSES

Line-Continuous Scanning Systems

$$g_r(x,y) = \int_{-\infty}^{\infty} p_W[x - x_s(t),\ y - y_s(t)]\ s(t)\ dt$$

$g_r(x,y)$ — reconstructed image

$p_W(x,y)$ — write spot

$s(t)$ — scan signal

$$s(t) = \tilde{g}[x_s(t),\ y_s(t)]$$

$$\tilde{g}(x,y) = p(-x,-y) ** g(x,y)$$

Recall

$$\tilde{g}_s(x,y) = q(x,y)\,\tilde{g}(x,y)$$

$$q(x,y) = \int_{-\infty}^{\infty} \delta[x - x_s(t),\ y - y_s(t)]\ dt$$

$$g_r(x,y) = \int_{-\infty}^{\infty} p_w[x - x_s(t),\ y - y_s(t)]\ \tilde{g}[x_s(t),\ y_s(t)]\ dt$$

$$= \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_w(x - \xi,\ y - \eta)\ \tilde{g}(\xi,\eta) \right.$$

$$\left. \times\ \delta[\xi - x_s(t),\ \eta - y_s(t)]\ d\xi\ d\eta \right\} dt$$

$$g_r(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_W(x-\xi,\ y-\eta)\ \tilde{g}(\xi,n)$$

$$\times \left\{ \int_{-\infty}^{\infty} \delta[\xi - x_s(t),\ \eta - y_s(t)]\ dt \right\} d\xi\ d\eta$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_W(x-\xi,\ y-\eta)\ \tilde{g}(\xi,\eta)\ q(\xi,\eta)\ d\xi\ d\eta$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_W(x-\xi,\ y-\eta)\ \tilde{g}_s(\xi,\eta)\ d\xi\ d\eta$$

$$= p_W(x,y) ** \tilde{g}_s(x,y)$$

Matrix-Addressable Systems

$$g_r(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} s_{mN+n} \; \mathbf{p_w}(\mathbf{x} - \mathbf{mX},\ \mathbf{y} - \mathbf{nY})$$

$$s_{mN+n} = \tilde{g}(mX,\ nY)$$

Recall

$$\tilde{g}_s(x,y) = q(x,y)\ \tilde{g}(x,y)$$

$$q(x,y) = \sum_m \sum_n \delta(x - mX,\ y - nY)$$

$$g_r(x,y) = \sum_m \sum_n \tilde{g}(mX,\ nY)\ p_W(x - mX,\ y - nY)$$

$$= \sum_m \sum_n \left\{ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{g}(\xi,\eta)\ p_W(x - \xi,\ y - \eta) \right.$$

$$\left. \times\ \delta(\xi - mX,\ \eta - nY)\ d\xi\ d\eta \right\}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{g}(\xi,\eta)\ p_W(x - \xi,\ y - \eta)$$

$$\times \left\{ \sum_m \sum_n \delta(\xi - mX,\ \eta - nY) \right\} d\xi\ d\eta$$

$$g_r(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{g}(\xi,\eta)\, p_W(x-\xi, y-\eta)\, q(\xi,\eta)\, d\xi\, d\eta$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_W(x-\xi,\, y-\eta)\, \tilde{g}_s(\xi,\eta)\, d\xi\, d\eta$$

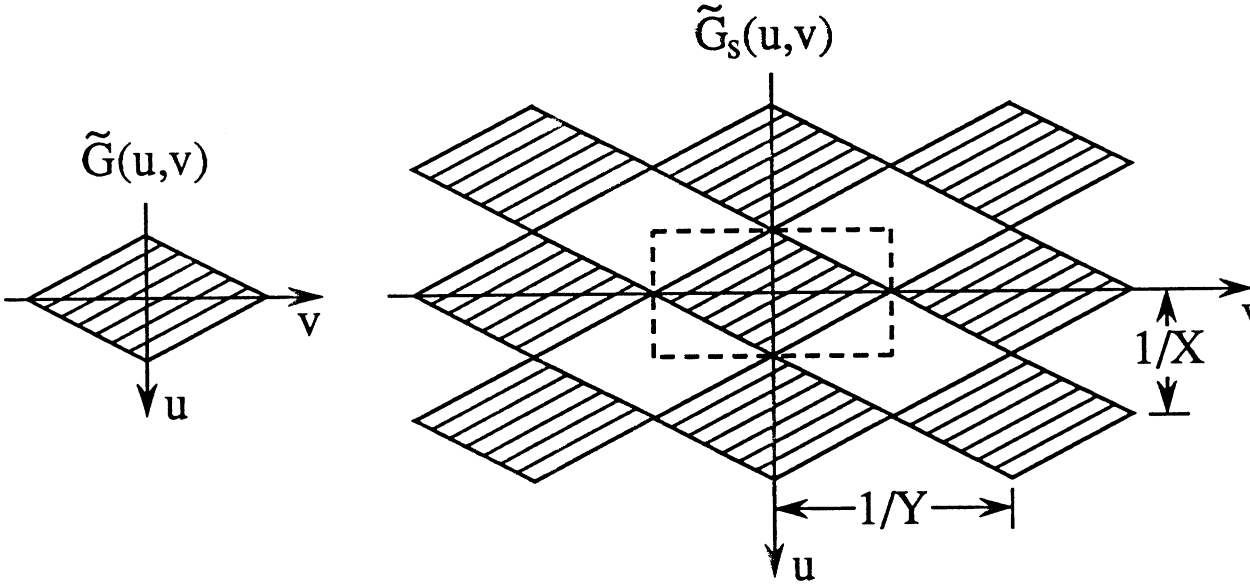
$$= p_W(x,y) ** \tilde{g}_s(x,y)$$

General Model

$$g_r(x,y) = p_w(x,y) ** \tilde{g}_s(x,y)$$

$$G_r(u,v) = P_w(u,v)\ \tilde{G}_s(u,v)$$

Ideal Reconstruction



$$\tilde{G}_S(u,v) = \frac{1}{XY} \sum_k \sum_l \tilde{G}(u - k/X,\ v - l/Y)$$

$$P_W(u,v) = XY\ \mathrm{rect}(Xu,\ Yv)$$

Spatial domain interpretation

$$p_W(x,y) = \text{sinc}\,(x/X,\ y/Y)$$

$$g_r(x,y) = p_W(x,y) ** \tilde{g}_s(x,y)$$

$$\tilde{g}_s(x,y) = q(x,y)\,\tilde{g}(x,y)$$

$$q(x,y) = \sum_m \sum_n \delta(x - mX,\ y - nY)$$

$$\tilde{g}_s(x,y) = \sum_m \sum_n \tilde{g}(mX,\ nY)\,\delta(x - mX,\ y - nY)$$

$$g_r(x,y) = \sum_m \sum_n \tilde{g}(mX,\ nY)\ \mathrm{sinc}\left(\frac{x - mX}{X}\ ,\ \frac{y - nY}{Y}\right)$$

$$g_r(kX,\ \ell Y) = \sum_m \sum_n \tilde{g}(mX,\ nY)\ \mathrm{sinc}\left(\frac{kX - mX}{X}\ ,\ \frac{\ell X - nY}{Y}\right)$$

$$= \sum_m \sum_n \tilde{g}(mX,\ nY)\ \mathrm{sinc}(k-m,\ \ell-n)$$

$$= \tilde{g}(kX,\ \ell Y)$$

If Nyquist conditions are satisfied, *i.e.* $\tilde{G}(u,v) = 0$ $|u| > 1/(2X)$, $|v| > 1/(2Y)$, then

$$g_r(x,y) \equiv \tilde{g}(x,y)$$

Zero Order Hold Reconstruction

$$p_w(x,y) = \mathrm{rect}(x/X,\ y/Y)$$

$$P_w(u,v) = XY\ \mathrm{sinc}(Xu,\ Xv)$$

$$G_r(u,v) = \mathrm{sinc}(Xu,\ Xv) \sum_k \sum_\ell \tilde{G}(u - k/X,\ v - \ell/Y)$$

$$\neq \tilde{G}(u,v)$$

Aliasing Artifacts

- due to presence of replications for which $(k,\ell) \neq (0,0)$ in spectrum of reconstructed image

  $$G_r(u,v) = P_w(u,v) \sum_k \sum_\ell \tilde{G}(u - k/X,\ v - \ell/Y)$$

- result in spurious low frequency patterns in displayed or printed image
  - moire patterns
  - jagged rendition of straight edges

Example illustrating moire formation during sampling of a sinewave grating