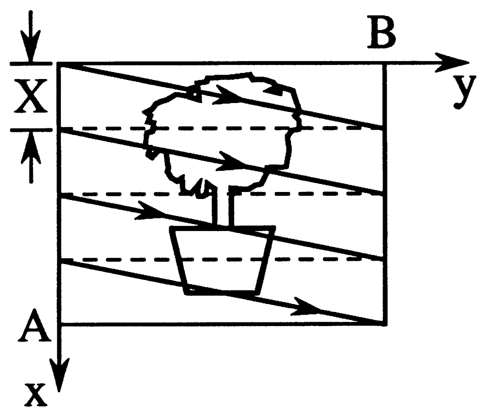
2.2.5 ANALYSIS OF SCANNING

Line-Continuous Scanning

Consider lexicographic scanning of a still image g(x,y)



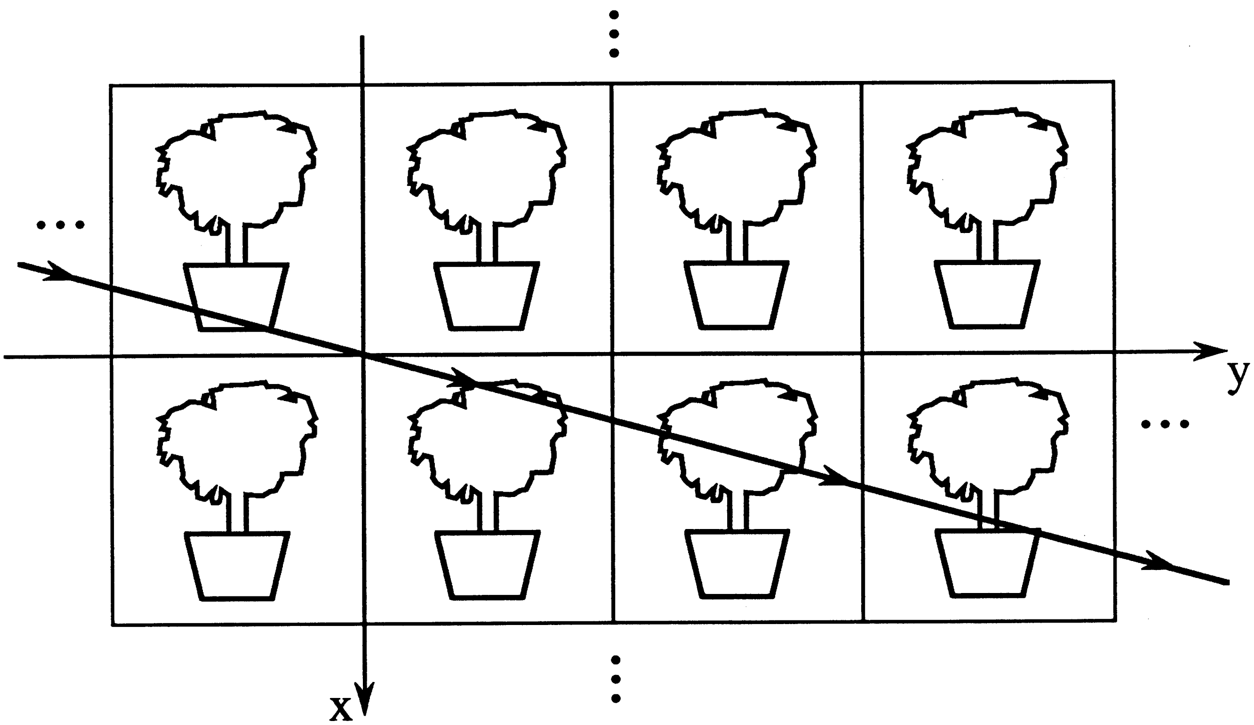
Assume: scan lines have slope B/X

line retrace is horizontal

$A/X = M$ an integer (number of scan lines)

During scanning of a single frame, scan line passes back and forth across field of view (FOV).

Achieve same effect by replicating the FOV and scanning along a straight line.



y
x

replicated image

$$g_p(x,y) = \text{rep}_{AB}[g(x,y)]$$

equation of scan line

$$ax + y = 0 \qquad a = -B/X$$

sampled image

$$g_s(x,y) = g_p(x,y)\delta(ax + y)$$

projection of sampled image onto x-axis

$$r(x) = \int_{-\infty}^{\infty} g_s(x,y)dy$$

conversion to function of time

$s(t) = r(Vt) \qquad V =$ velocity of scan beam along x axis

Fourier Analysis of Line-Continuous Scanning

$$s(t) = r(Vt)$$

$$S(f) = \frac{1}{V} R \left(\frac{f}{V}\right)$$

$$r(x) = \int_{-\infty}^{\infty} g_s(x,y)dy$$

$$R(u) = \int_{-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} g_s(x,y)dy \right\} e^{-j2\pi ux} dx$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_s(x,y) e^{-j2\pi(ux+0y)} dxdy$$

$$= G_s(u,0)$$

$$g_s(x,y) = g_p(x,y)\ \delta(ax+y)$$

$$= g_p(x,y)\ d(x,y)$$

$$G_s(u,v) = G_p(u,v) ** D(u,v)$$

$$D(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta(ax+y)e^{-j2\pi(ux+vy)}dxdy$$

$$= \int_{-\infty}^{\infty} e^{-j2\pi[ux+v(-ax)]}dx$$

$$= \int_{-\infty}^{\infty} e^{-j2\pi(u-av)x}dx$$

$$= \delta(u-av)$$

$$G_s(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_p(u-\mu, v-\nu)\, D(\mu,\nu)\, d\mu\, d\nu$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G_p(u-\mu, v-\nu)\, \delta(\mu - a\nu)\, d\mu\, d\nu$$

$$= \int_{-\infty}^{\infty} G_p(u - a\nu, v-\nu)\, d\nu$$

$$g_p(x,y) = \mathrm{rep}_{AB}[g(x,y)]$$

$$G_p(u,v) = \frac{1}{AB}\, \mathrm{comb}_{1/A\ 1/B}\, [G(u,v)]$$

$$= \frac{1}{AB} \sum_k \sum_\ell G(k/A,\ \ell/B)\, \delta(u - k/A, v - \ell/B)$$

$$R(u) = G_s(u,0)$$

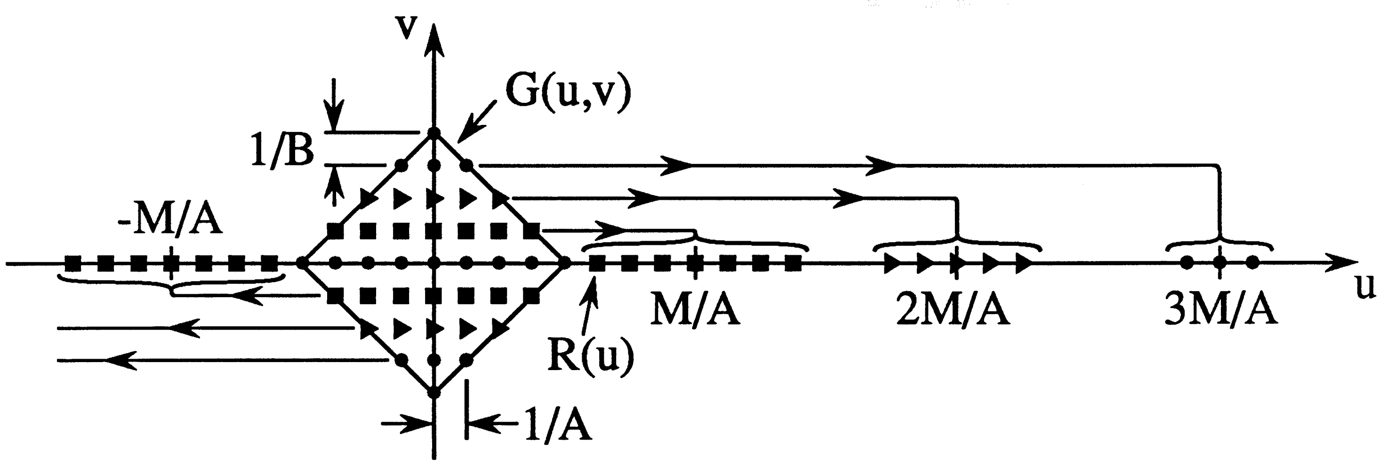
$$= \frac{1}{AB} \sum_k \sum_\ell G(k/A, \ell/B)\ \delta(u + \ell a/B - k/A)$$

$$= \frac{1}{AB} \sum_k \sum_\ell G(k/A, \ell/B)\ \delta[u - (\ell M + k)/A]$$

since $a = -B/X$ and $1/X = M/A$

Interpretation

$$R(u) = \frac{1}{AB} \sum_k \sum_\ell G(k/A, \ell/B)\, \delta[u - (\ell M + k)/A]$$



Spectral groups will not overlap provided $G(u,v) = 0$, $|u| > M/(2A) = 1/(2X)$.

This is the Nyquist condition that was derived earlier.

Spectrum of Scanned Signal

$$S(f) = \frac{1}{V} R\left(\frac{f}{V}\right)$$

$$= \frac{1}{ABV} \sum_k \sum_\ell G(k/A, \ell/B)\, \delta[f/V - (\ell M + k)/A]$$

Recall identity

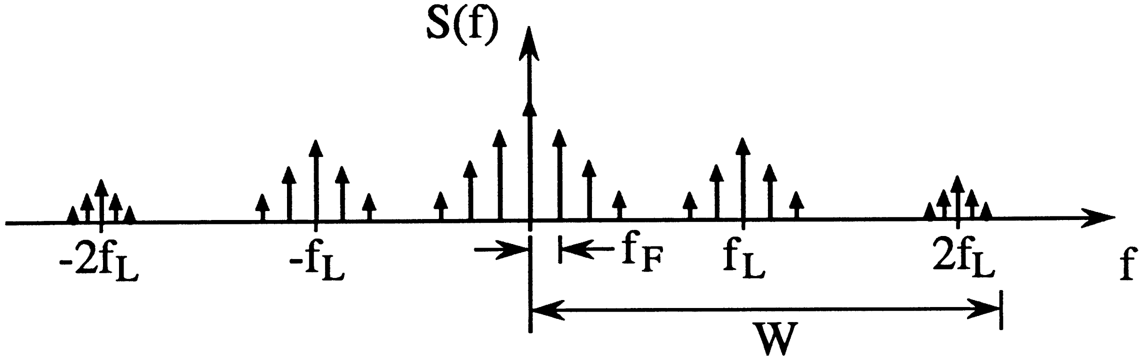
$$\delta(ax + b) = \frac{1}{|a|}\, \delta\left(x + \frac{b}{a}\right)$$

$$S(f) = \frac{1}{AB} \sum_k \sum_\ell G(k/A, \ell/B)\, \delta[f - (\ell M + k)(V/A)]$$

Frame period $T_F = A/V = 1/f_F$

Line period $T_L = T_F/M = 1/f_L$

$$S(f) = \frac{1}{AB} \sum_k \sum_\ell G(k/A, \ell/B)\, \delta\left[f - (\ell f_L + k f_F)\right]$$



Example (NTSC Video)

$$f_F = 30\,\text{Hz} \quad M = 500 \quad f_L = 15\ \text{kHz}$$

Maximum spatial frequency along y axis $= M/(2B)$
$\Rightarrow W = 3.75$ MHz

Now combine everything

$$G_s(u,v) = \int_{-\infty}^{\infty} G_p(u-a\nu, v-\nu)d\nu$$

$$= \int_{-\infty}^{\infty} \left\{ \frac{1}{AB} \sum_k \sum_\ell G(k/A, \ell/B)\, \delta(u-a\nu-k/A,\ v-\nu-\ell/B) \right\} d\nu$$

$$= \frac{1}{AB} \sum_k \sum_\ell G(k/A, \ell/B) \int_{-\infty}^{\infty} \delta(u-a\nu-k/A)\delta(v-\nu-\ell/B)d\nu$$

$$= \frac{1}{AB} \sum_k \sum_\ell G(k/A, \ell/B)\, \delta[u-a(v-\ell/B)-k/A]$$

Spectral Mappings

- High vertical spatial frequencies in G(u,v) are mapped to edge of each spectral group.
- High horizontal spatial frequencies in G(u,v) are mapped to the higher index spectral groups.

Extensions to the Analysis

- 2:1 line-interlaced scanning
  - proper choice of model parameters
- Scanning along horizontal lines
  - shift each succeeding column of replications of $g(x,y)$ up by X to obtain $g_p(x,y)$
  - $r(y) = g_p(0,y)$
  - results are essentially the same as those that we obtained

- Scanning of time-varying imagery
  - replicate g(x,y,t) in (x,y) with period (A,B) to obtain $g_p(x,y,t)$
  - tilt scan line out along time axis

$$g_s(x,y,t) = g_p(x,y,t)\,\delta(ax+t,\ by+t)$$

  - project onto time axis

$$s(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g_s(x,y,t)\,dx\,dy$$

  - results are similar to those obtained with still imagery; each spectral line is spread into a profile representing effect of time variation.

- Horizontal and vertical blanking interval
  - replicate $g(x,y,t)$ in (x,y) with period (A',B') where $A' > A$ and $B' > B$.

Dot-Interlaced Scanning

- Motivation
  - — reduce flicker due to phosphor decay at display
  - — improve resolution of high spatiotemporal frequency components
- Model

$$g_S(x,y,t) = q(x,y,t)\; g(x,y,t)$$

$$q(x,y,t) = \sum_{k=-\infty}^{\infty} \delta(x-\alpha_k X,\; y-\beta_k Y,\; t-kT_S)$$

T_S – sampling interval

(α_k, β_k) – sampling pattern

Properties of the Sampling Pattern

- FOV is $M \times N$
- (α_k, β_k), $k = 0, \ldots MN - 1$ is a permutation of the integer pairs (a, b), $0 \leq a \leq M-1$ and $0 \leq b \leq N-1$
- Sampling pattern repeats from frame to frame

$$\left(\alpha_{k+cMN}, \beta_{k+cMN}\right) = \left(\alpha_k, \beta_k\right) \quad \text{for all integers } c$$

- frame interval $T_F = MNT_S$

SAMPLING PATTERNS

Conventional Patterns:

Lexicographic

0	1	2	3	4	5	6	7
8	9	10	11	12	13	14	15
16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

2:1 Line Interlaced

0	1	2	3	4	5	6	7
32	33	34	35	36	37	38	39
8	9	10	11	12	13	14	15
40	41	42	43	44	45	46	47
16	17	18	19	20	21	22	23
48	49	50	51	52	53	54	55
24	25	26	27	28	29	30	31
56	57	58	59	60	61	62	63

Examples

- Lexicographic

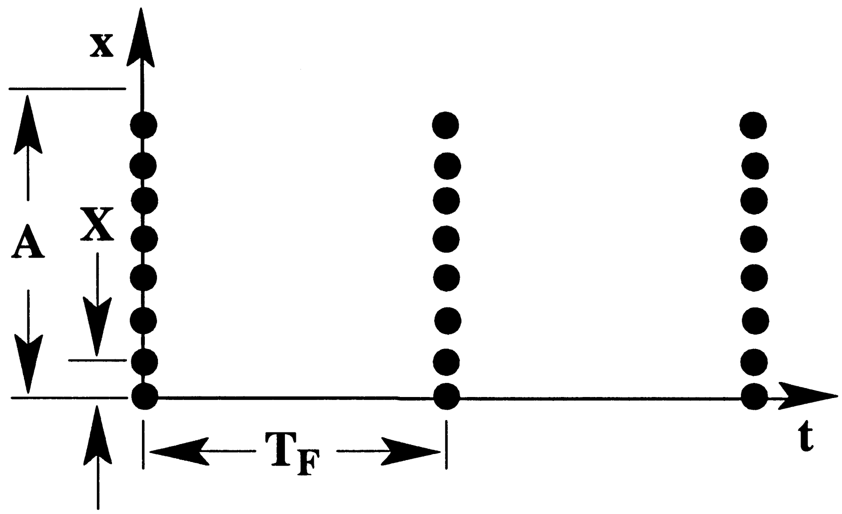
$$\beta_k = k \bmod N$$
$$\alpha_k = \lfloor k/N \rfloor \bmod M$$

- 2:1 line-interlaced

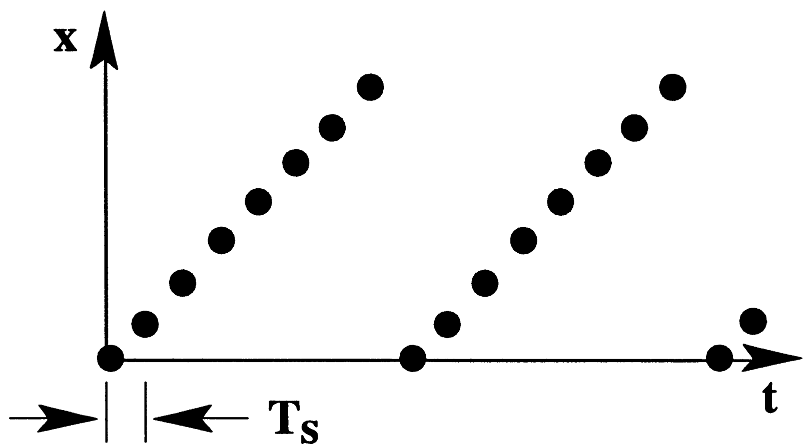
$$\beta_k = k \bmod N$$

$$\alpha_k = \begin{cases} 2\lfloor k/N \rfloor \bmod M/2, & \lfloor k/N \rfloor \bmod M \le M/2-1 \\ 2\lfloor k/N \rfloor \bmod M/2+1, & \lfloor k/N \rfloor \bmod M \ge M/2 \end{cases}$$

FRAME-INSTANTANEOUS SAMPLING



TIME-SEQUENTIAL SAMPLING
Lexicographic Pattern



Novel Patterns:

Bit Reversed

0	32	8	40	2	34	10	42
48	16	56	24	50	18	58	26
12	44	4	36	14	46	6	38
60	28	52	20	62	30	54	22
3	35	11	43	1	33	9	41
51	19	59	27	49	17	57	25
15	47	7	39	13	45	5	37
63	31	55	23	61	29	53	21

$$M = 2^m, N = 2^n$$

Congruential

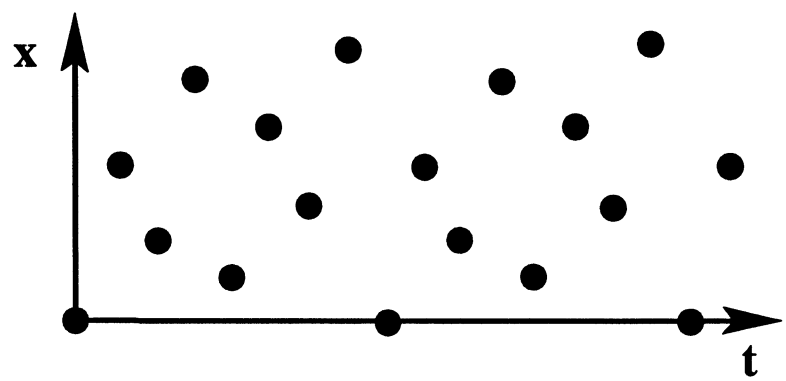
0	21	42	7	28	49	14	35
24	45	10	31	52	17	38	3
48	13	34	55	20	41	6	27
16	37	2	23	44	9	30	51
40	5	26	47	12	33	54	19
8	29	50	15	36	1	22	43
32	53	18	39	4	25	46	11

- Congruential

$$\alpha_k = \alpha_1 \text{ k mod M}$$
$$\beta_k = \beta_1 \text{ k mod N}$$

$$\gcd(\alpha_1, M) = 1$$
$$\gcd(\beta_1, N) = 1$$
$$\gcd(M, N) = 1$$

BIT-REVERSED PATTERN



Spectral Analysis of Dot-Interlaced Scanning

$$G_S(u,v,f) = \frac{1}{XYT_F} \sum_m \sum_n \sum_p Q_{mnp} G(u-m/A, v-n/B, f-p/T_F)$$

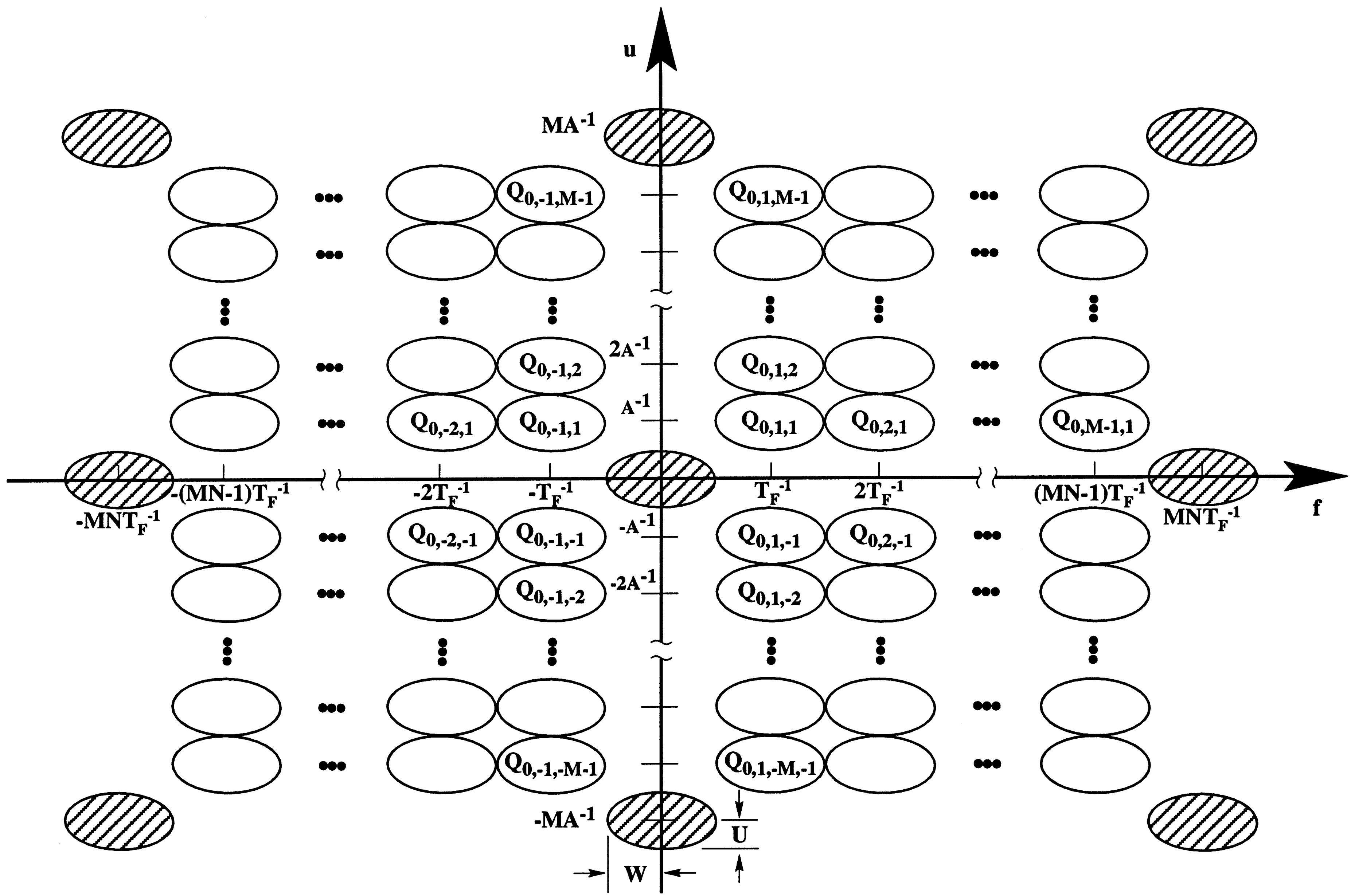
where

$$Q_{mnp} = \frac{1}{MN} \sum_{k=0}^{MN-1} e^{-j2\pi(\alpha_k m/M + \beta_k n/N + pk/MN)}$$

Properties of the coefficients

$$Q_{000} = 1$$
$$Q_{mn0} = \delta_m \bmod M \quad \delta_n \bmod N$$
$$Q_{00p} = \delta_p \bmod MN$$

SPECTRUM OF TIME-SEQUENTIALLY SAMPLED SIGNAL



Spectral characteristics of the sampling pattern examples

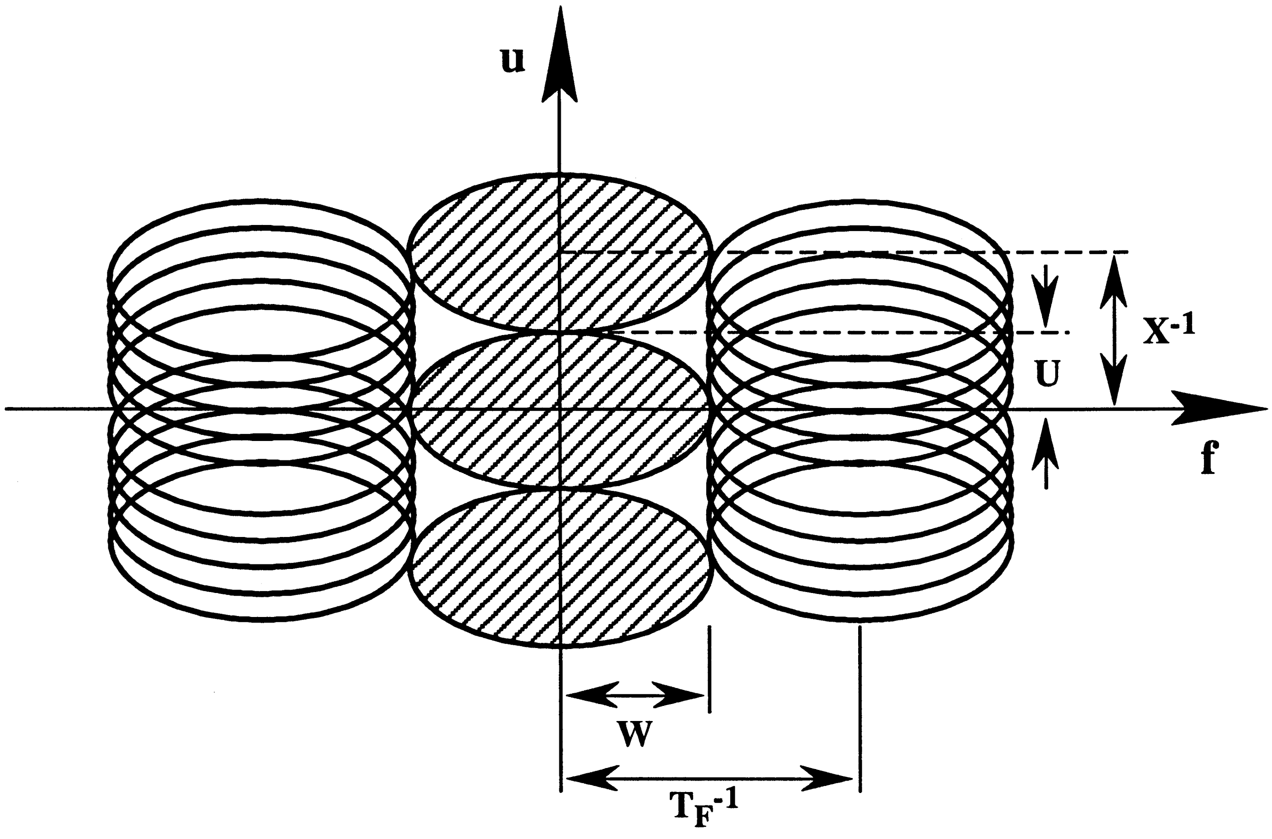
- Lexicographic

$$| Q_{\pm 1, 0, \pm 1} | \cong 1$$

- 2:1 line-interlaced

$$| Q_{\pm 1, 0, \pm 2} | \cong 1$$

SPECTRUM UNDER WORST CASE NYQUIST CONDITIONS



- Bit reversed

$$
|Q_{mnp}| \le \begin{cases} 2\pi \min(m^2, n^2)\,|p|/(MN), & 0<|m|<M, \\ & 0<|n|<N \\ 2\pi m^2\,|p|/(MN), & 0<|m|<M,\ n=0 \\ 2\pi n^2\,|p|/(MN), & m=0,\ 0<|n|<N \end{cases}
$$

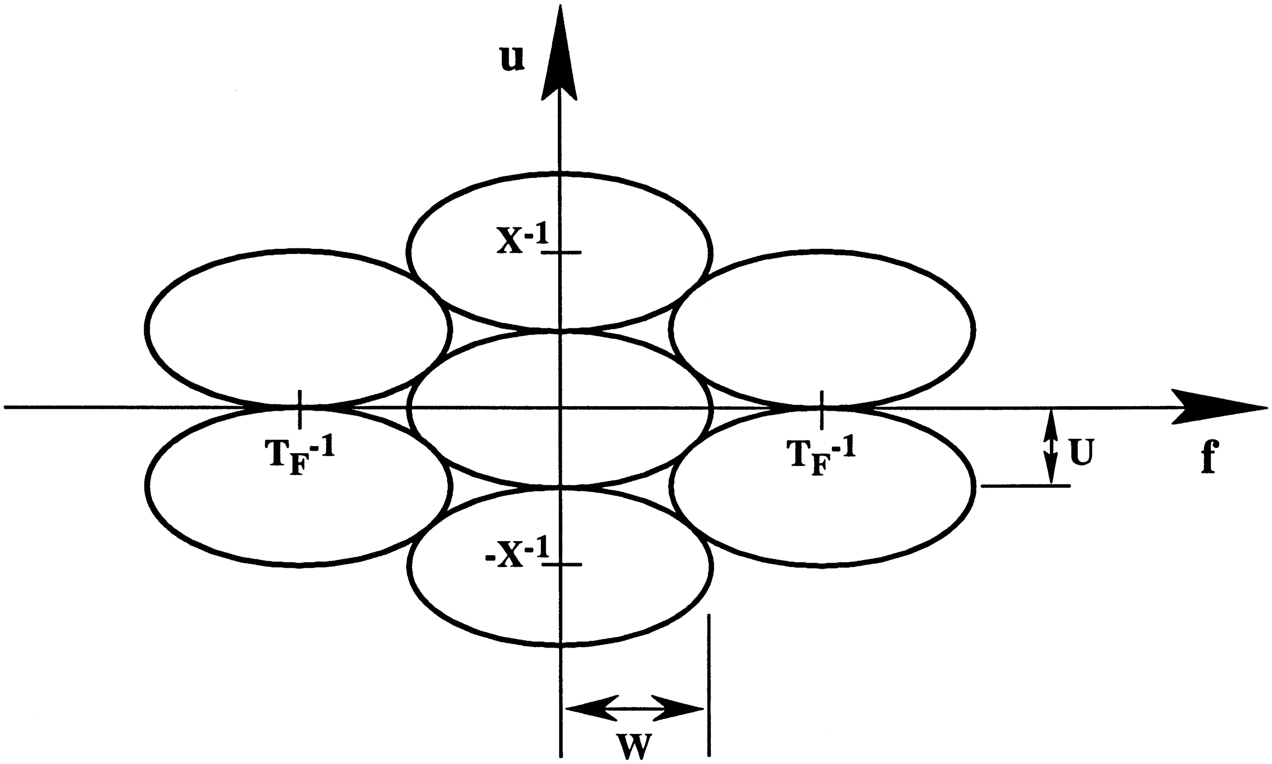
- Congruential

$$
Q_{mnp} = \delta_{m-\tilde{\alpha}_p}\ \delta_{n-\tilde{\beta}_p}
$$

where $(\tilde{\alpha}_p, \tilde{\beta}_p)$ is the dual congruential sampling pattern

$$
\begin{aligned} \tilde{\alpha}_p &= \tilde{\alpha}_1 p \bmod M \\ \tilde{\beta}_p &= \tilde{\beta}_1 p \bmod N \end{aligned}
$$

SPECTRUM UNDER NYQUIST CONDITIONS WITH OPTIMAL SAMPLING PATTERN



Evaluation of sampling patterns

- Assume a signal model

  — g(x,y,t) band limited to hyperellipsoid

$$\Omega = \left\{ (u,v,f)\colon (u/U)^2 + (v/U)^2 + (f/W)^2 \le 1 \right\}$$

  — g(x,y,t) wide-sense stationary stochastic process with power spectral density

$$S_{gg}(u,v,f) = \begin{cases} (3\sigma_g^2)/4\pi U^2 W), & (u,v,f) \in \Omega \\ 0, \text{ else} \end{cases}$$

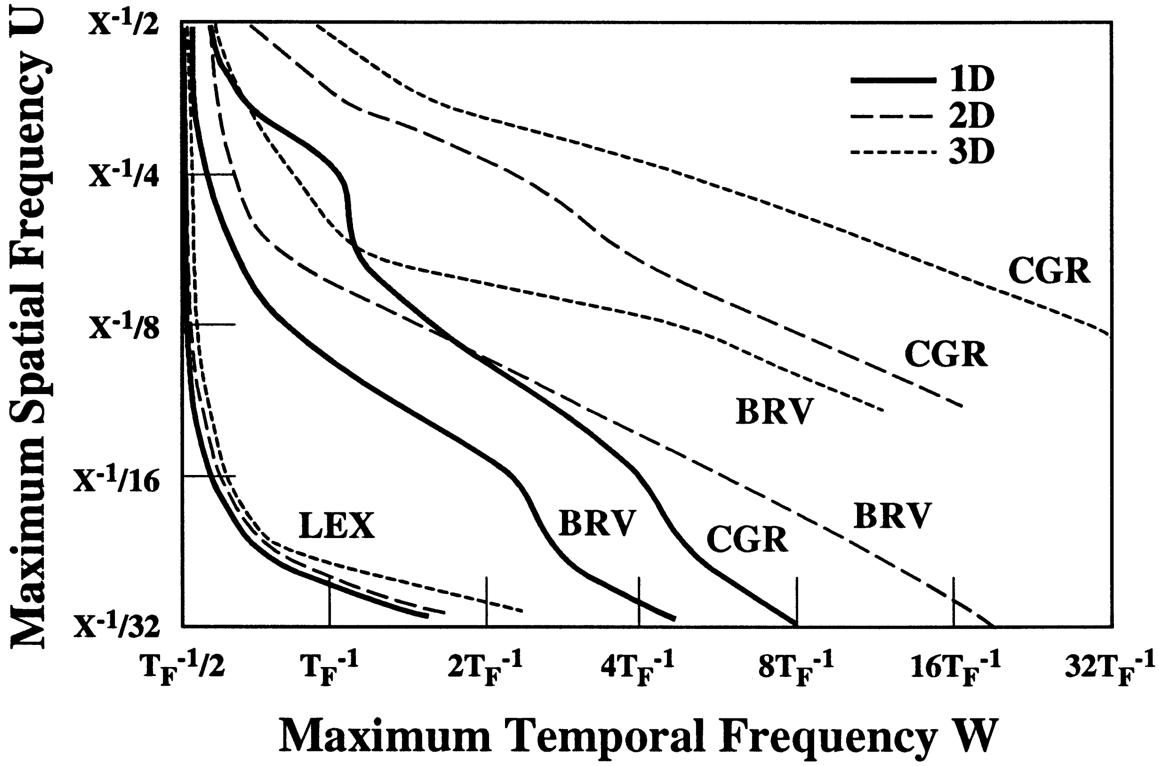
- Nyquist rate
  - fix X and Y
  - increase T_s until overlap of spectra occurs

- Signal-to-aliasing noise power ratio

$$e(x,y,t) = LPF_{\Omega}\{g_s(x,y,t) - g(x,y,t)\}$$

$$\Phi = \sigma_g^2 / \sigma_e^2$$

TRADEOFF BETWEEN MAXIMUM RESOLVABLE SPATIAL AND TEMPORAL FREQUENCIES†



† ellipsoidally bandlimited signals sampled at 30dB SNR