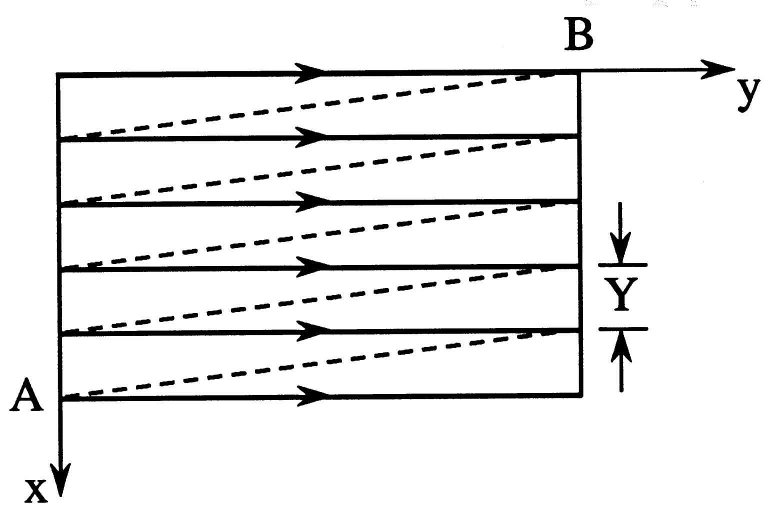
2.2.3 ANALYSIS OF SAMPLING

Line-Continuous Scanning

$$q(x,y) = \int_{-\infty}^{\infty} \delta[x - x_s(t),\ y - y_s(t)]\,dt$$



$$q(x,y) = \sum_{m} \delta(x - mX)$$

$$= \mathrm{rep}_X[\delta(x)] \cdot 1$$

$$Q(u,v) = \frac{1}{X}\,\mathrm{comb}\,\frac{1}{X}[1] \cdot \delta(v)$$

$$= \frac{1}{X} \sum_{k} \delta(u - k/X)\ \delta(v)$$

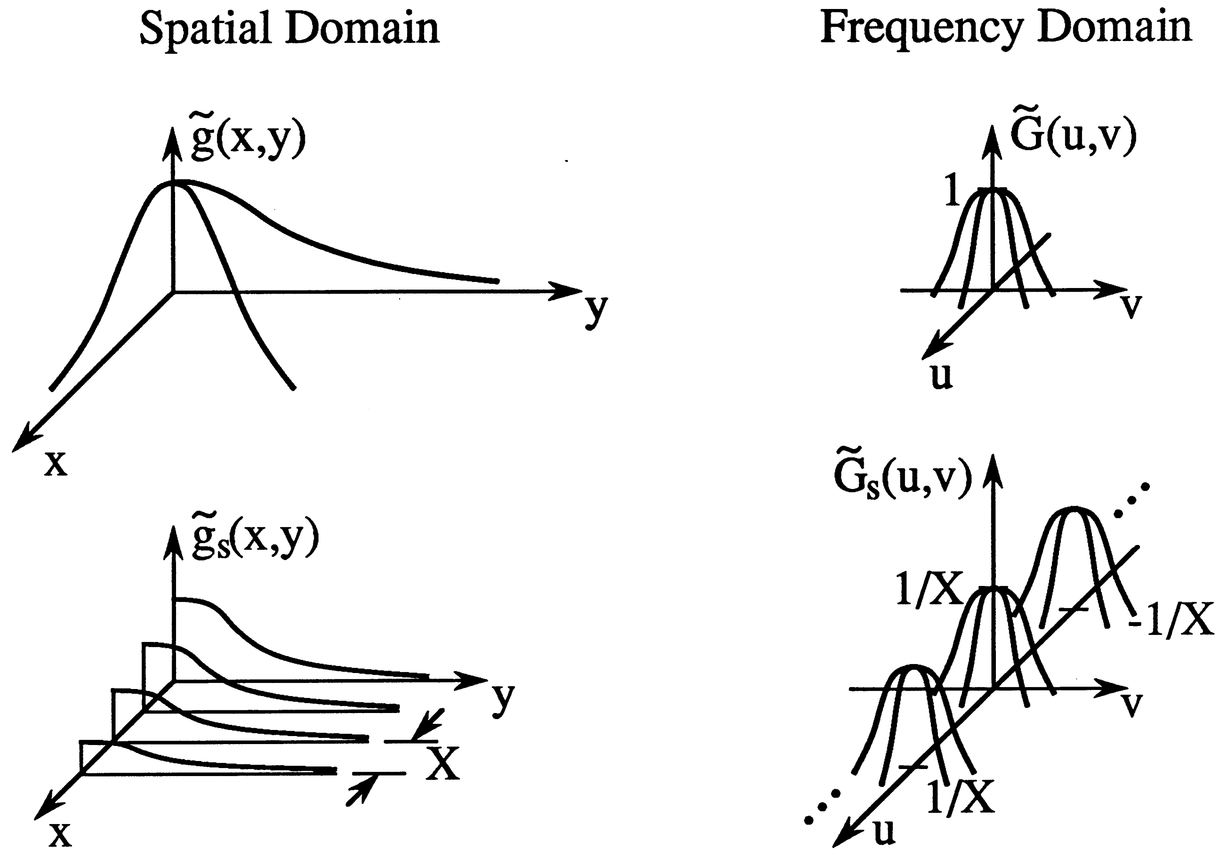
$$= \frac{1}{X} \sum_{k} \delta(u - k/X, v)$$

$$\tilde{g}_s(x,y) = q(x,y)\tilde{g}(x,y)$$

$$G_s(u,v) = Q(u,v) ** \tilde{G}(u,v)$$

$$= \left[\frac{1}{X} \sum_k \delta(u - k/X, v)\right] ** \tilde{G}(u,v)$$

$$= \frac{1}{X} \sum_k \tilde{G}(u - k/X, v)$$



Spatial Domain
Frequency Domain
g̃(x,y)
y
x
G̃(u,v)
1
v
u
g̃s(x,y)
y
X
x
G̃s(u,v)
1/X
-1/X
v
1/X
u

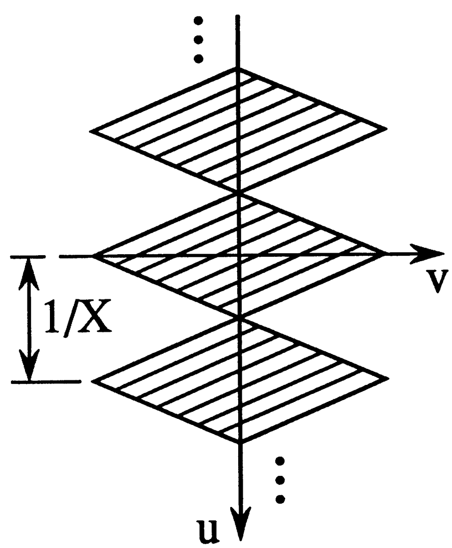
Nyquist condition for line-continuous scanning

The aperture smoothed image $\tilde{g}(x,y)$ may be uniquely reconstructed from its line-continuous scanned version $\tilde{g}_S(x,y)$ provided

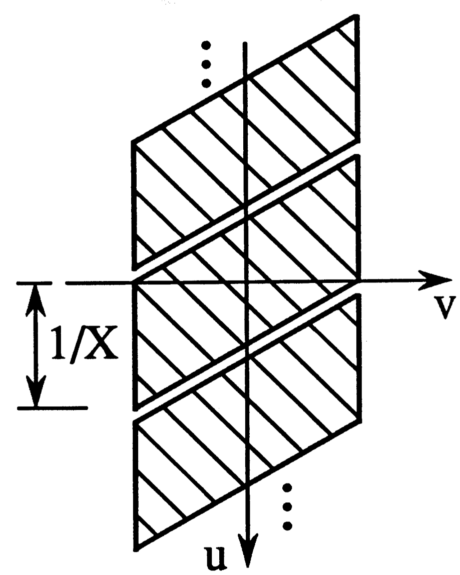
$$\tilde{G}(u,v) = 0, \qquad |u| > 1/(2X)$$

Note that this condition is sufficient but not necessary for perfect reconstruction.

Perfect reconstruction is possible in both cases shown below.

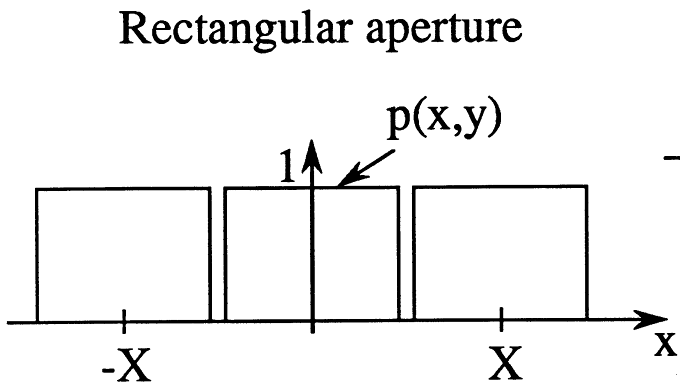


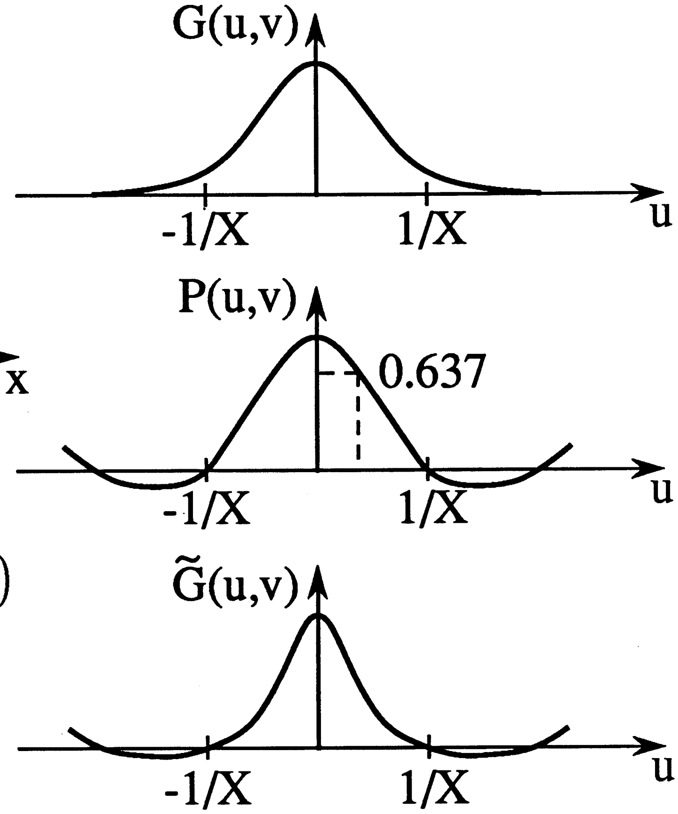
Satisfies Nyquist criterion



does not satisfy Nyquist criterion

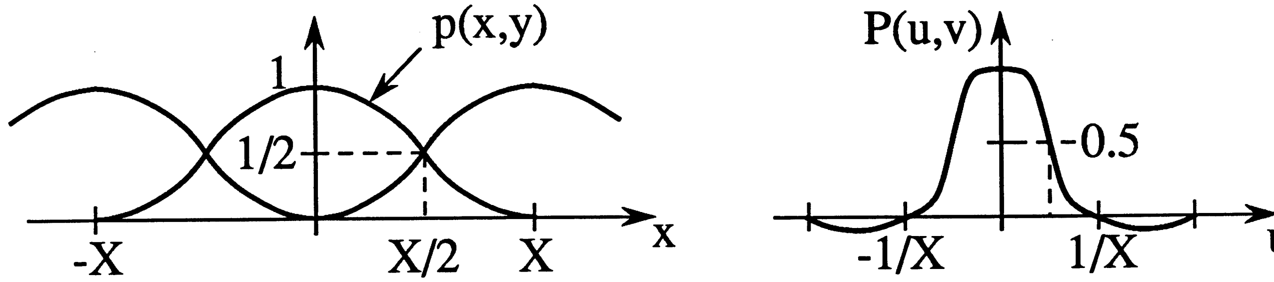
Pre-Scan Bandlimiting Effect of Aperture





$$\tilde{g}(x,y) = p(-x,-y) ** g(x,y)$$
$$\tilde{G}(u,v) = P(-u,-v)G(u,v)$$

Raised cosine aperture



Sampling Effects with Focal Plane Arrays

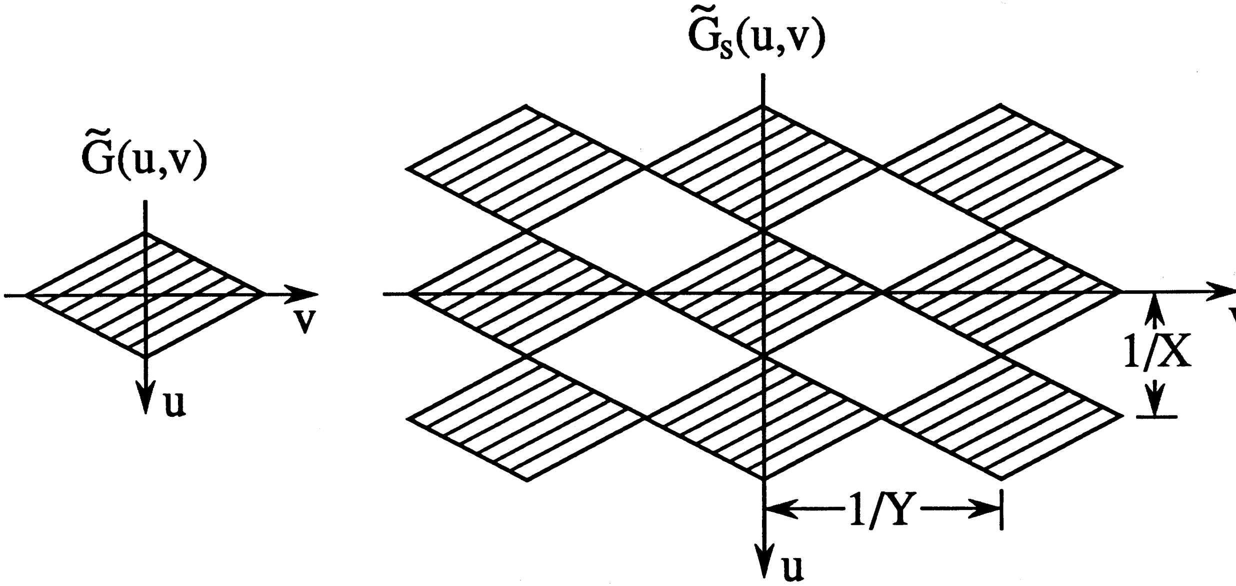
$$q(x,y) = \sum_m \sum_n \delta(x - mX, y - nY)$$

$$\tilde{g}_s(x,y) = q(x,y)\tilde{g}(x,y)$$

$$= \mathrm{comb}_{XY}[\tilde{g}(x,y)]$$

$$\tilde{G}_s(u,v) = \frac{1}{XY}\mathrm{rep}_{\frac{1}{X}\frac{1}{Y}}[\tilde{G}(u,v)]$$

$$= \frac{1}{XY} \sum_k \sum_\ell \tilde{G}(u - k/X, v - \ell/Y)$$



Nyquist condition for 2-D sampling on a rectangular lattice

The aperture smoothed image $\tilde{g}(x,y)$ may be uniquely reconstructed from its sampled version $\tilde{g}_S(x,y)$ provided

$$\tilde{G}(u,v) = 0, \quad |u| > 1/(2X) \text{ and } |v| > 1/(2Y)$$

Again, condition is sufficient but not necessary.