2.2.2 DEVELOPMENT OF A GENERAL MODEL

Line-Continuous Flying Spot Process

$$s(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_i[\xi - x_s(t),\ \eta - y_s(t)]p_r[\xi - x_s(t),\ \eta - y_s(t)] \times g(\xi,\eta)\, d\xi\, d\eta$$

$p_i(x,y)$ – illuminating spot profile

$pr(x,y)$ – read spot profile

$[x_s(t),\ y_s(t)]$ – scan trajectory

$g(x,y)$ – continuous-space still image

$s(t)$ – scan signal

- Combine illuminating and read spot profiles as one function

$$p(x,y) = p_i(x,y)\, p_r(x,y)$$

- Separate the three processes:
  - integration over aperture
  - sampling
  - scanning (consider this later)

Integration over Aperture

$$s(t) = \int \int p[\xi - x_s(t),\ \eta - y_s(t)]\ g(\xi,\eta)\ d\xi\, d\eta$$

define

$$\tilde{g}(x,y) = \int \int p[\xi - x,\ \eta - y]\ g(\xi,\eta)\ d\xi\, d\eta$$

$$= \int \int p[-(x-\xi),\ -(y-\eta)]\ g(\xi,\eta)\ d\xi\, d\eta$$

$$= p(-x,\ -y) ** g(x,y)$$

then

$$s(t) = \tilde{g}\ [x_s(t),\ y_s(t)]$$

Sampling

define

$$q(x,y) = \int_{-\infty}^{\infty} \delta[x - x_s(t),\ y - y_s(t)]\ dt$$

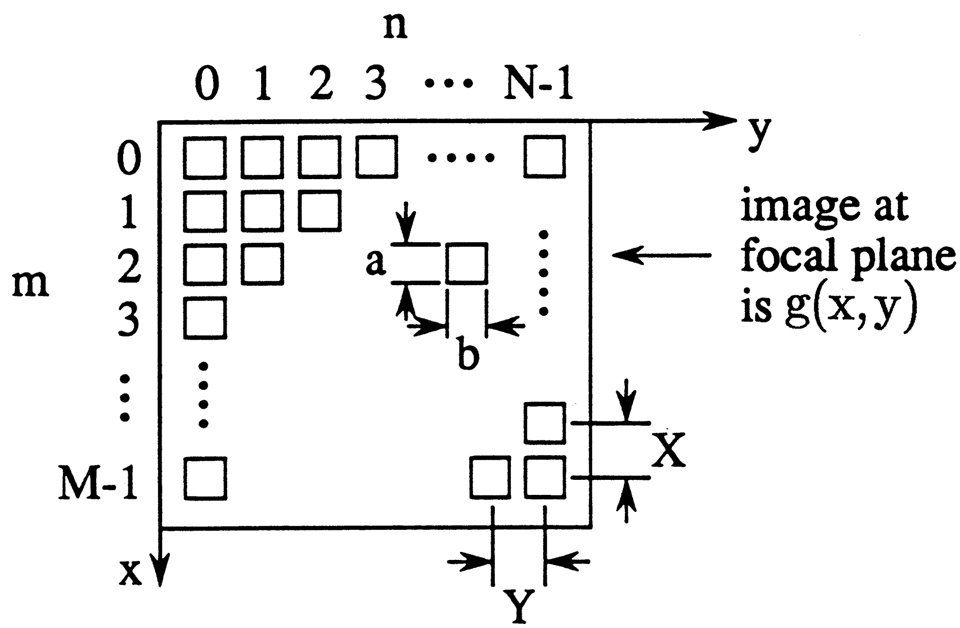
then let

$$\tilde{g}_s(x,y) = q(x,y)\ \tilde{g}(x,y)$$

This signal embodies all the effects due to the fact that we only see $\tilde{g}(x,y)$ along the locus of points $[x_s(t),\ y_s(t)],\ -\infty < t < \infty$.

With regard to these sampling effects, it is unimportant how we map the signal information into a 1-D function of time.

Focal Plane Array Scan Process



$$s_{mN+n} = \int_{mX-a/2}^{mX+a/2} \int_{nY-b/2}^{nY+b/2} g(\xi,\eta)\, d\xi\, d\eta$$

Integration over Aperture

$$s_{mN+n} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{rect}\left(\frac{\xi - mX}{a}, \frac{\eta - nY}{b} \right) g(\xi, \eta) d\xi \, d\eta$$

let

$$p(x,y) = \mathrm{rect}\left(\frac{x}{a}, \frac{y}{b}\right)$$

define

$$\tilde{g}(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(\xi - x, \eta - y) \, g(\xi, \eta) d\xi \, d\eta$$

$$= p(-x, -y) ** g(x,y)$$

then

$$s_{mN+n} = \tilde{g}(mX, nY)$$

Sampling

define

$$q(x,y) = \sum_{m} \sum_{n} \delta(x-mX,\ y-nY)$$

then let

$$\tilde{g}_s(x,y) = q(x,y)\ \tilde{g}(x,y)$$

Again, this signal embodies all the effects due to the fact that we observe g(x,y) only at locations (mX,nY)

General Model for Scanning and Sampling

Aperture effects

$$\tilde{g}(x,y) = p(-x, -y) ** g(x,y)$$

$$\tilde{G}(u,v) = P(-u, -v)\, G(u,v)$$

- Aperture acts as a filter
- As p(x,y) spreads out, P(u,v) contracts resulting in attenuation of the higher frequency components of the image g(x,y).

Sampling effects

$$\tilde{g}_S(x,y) = q(x,y)\,\tilde{g}(x,y)$$

$$\tilde{G}_S(u,v) = Q(u,v) ** \tilde{G}(u,v)$$

- Since q(x,y) generally contains periodic structures, Q(u,v) will consist of an array of impulses.
- Convolution of Q(u,v) with $\tilde{G}(u,v)$ will result in replications of $\tilde{G}(u,v)$ located at the coordinates of each impulse in Q(u,v).